CALCULATING THE UNCERTAINTY OF A DERIVED UNIT

Up to now, we have estimated experimental uncertainties as follows.

EXAMPLES: 55 ± 1 s   14.35 ± 0.01 g   15.2 ± 0.1 mL

These are called ***ABSOLUTE*** uncertainties because they tell us the limits within which we believe the experimental value lies.

Recall that **derived units** are created by dividing and/or multiplying units. Calculating the uncertainties associated with derived units requires us to use a special set of rules.

The Rules for Calculating the Uncertainties of Derived Units

1. Convert the absolute uncertainties of each measurement to a percentage uncertainty.
2. Add the individual percentage uncertainties to obtain the total percentage uncertainty.

EXAMPLE: Calculate the uncertainty in the density of a crystal when the mass of the crystal is 3.52 ± 0.01 g and the volume is 1.34 ± 0.02 mL. Quartz crystals have densities which lie in the range 2.635 to 2.660 g/mL. Based only on the calculated density, could the crystal be quartz?

First, calculate the value of the density (without using the uncertainties):

$$d = \frac{m}{V} = \frac{3.52\text{ g}}{1.34\text{ mL}} = 2.63\text{ g/mL}$$

Next, express each uncertainty as a percentage of its estimated value:

$$\frac{0.01\text{ g}}{3.52\text{ g}} \times 100\ \% = 0.28\ \% \quad \text{and} \quad \frac{0.02\text{ mL}}{1.34\text{ mL}} \times 100\ \% = 1.49\ \%$$

(Note that the units cancel and the percentages have no units.)

Now, add the percentage uncertainties to obtain the total percentage uncertainty:

0.28 % + 1.49 % = 1.77 %

Finally, round off the total percentage uncertainty to ONE significant digit and attach the percentage uncertainty:

$$d = \frac{3.52 \pm 0.01\text{ g}}{1.34 \pm 0.02\text{ mL}} = \mathbf{2.63\ g/mL \pm 2\ \%}$$

Note that the percentage uncertainty is added at the end, following the value and units. Because the absolute uncertainties are only stated with one significant figure, the percentage uncertainty also is only given to one significant figure.

In order to compare the calculated value to the experimentally-accepted range of values, we must convert the percentage uncertainty to an absolute uncertainty:

$$2\ \%\text{ of } 2.63\text{ g/mL} = \frac{2}{100} \times 2.63\text{ g/mL} = 0.05\text{ g/mL}$$

This means the calculated value has a range of (2.63 – 0.05 = 2.58) g/mL to (2.63 + 0.05 = 2.68) g/mL. The accepted values lie within this range and hence the crystal could be quartz.

EXAMPLE: The top of a bench has a length of 2.25 ± 0.01 m and a width of 0.75 ± 0.01 m. Calculate the uncertainty in the area of the bench top.

Initial calculation: Area = length x width = 2.25 m x 0.75 m = 1.7 m^2

Next, calculate the individual percentages:

$$\frac{0.01}{2.25} \times 100\ \% = 0.44\ \% \quad \text{and} \quad \frac{0.01}{0.75} \times 100\ \% = 1.33\ \%$$

Finally, add the percentages, round the result to one digit and attach the percentage to the calculation:

Area = **1.7 m^2 ± 2 %**

EXERCISES:

1. A student found that 14.52 ± 0.01 g of a chemical reacted at a constant rate and was completely consumed in 38.6 ± 0.5 s. Calculate the rate of the reaction, in grams/second, and the percentage uncertainty of the rate.

2. An astronomer measures the distance travelled by a comet, using the formula: distance = velocity x time. She determines that the comet is travelling at 58.6 ± 0.9 km/s. Calculate the distance, in kilometers, the comet travels during an observation period of 18.6 ± 0.2 s, and calculate the absolute uncertainty in the distance.

3. An automobile travels 435 ± 1 km and uses 32.6 ± 0.4 L of fuel. Calculate the rate of fuel consumption, in L/km, and the absolute uncertainty in the rate.

4. A section of main line railway track has a mass to length ratio of 64.5 ± 0.5 kg/m. A demonstration uses a piece of main line track having a length of 15.0 ± 0.2 cm. Calculate the mass of the small piece of track and the absolute uncertainty.

5. A flask is marked to contain "500 ± 5 mL". A student used an inexpensive household balance that was accurate to ± 1 g to measure out 15 g of salt, pour it into the half-filled flask, shake the mixture until the salt dissolved and fill the flask to the 500 mL mark. Calculate the concentration of the salt solution, in grams/litre, and the absolute uncertainty in the concentration.

6. Thin magnesium ribbon is frequently used in student experiments. However, short lengths of the ribbon have such a small mass that a precise value cannot be found on a typical centigram balance. Therefore, the mass of a short piece of magnesium ribbon is estimated as follows. A long piece of the ribbon has a length of 100.0 ± 0.1 cm and a mass of 3.84 ± 0.01 g. A piece of the ribbon, having a length of 2.50 ± 0.02 cm is reacted in an experiment to produce hydrogen gas. Calculate the mass of the smaller length of ribbon and the absolute uncertainty in its mass.