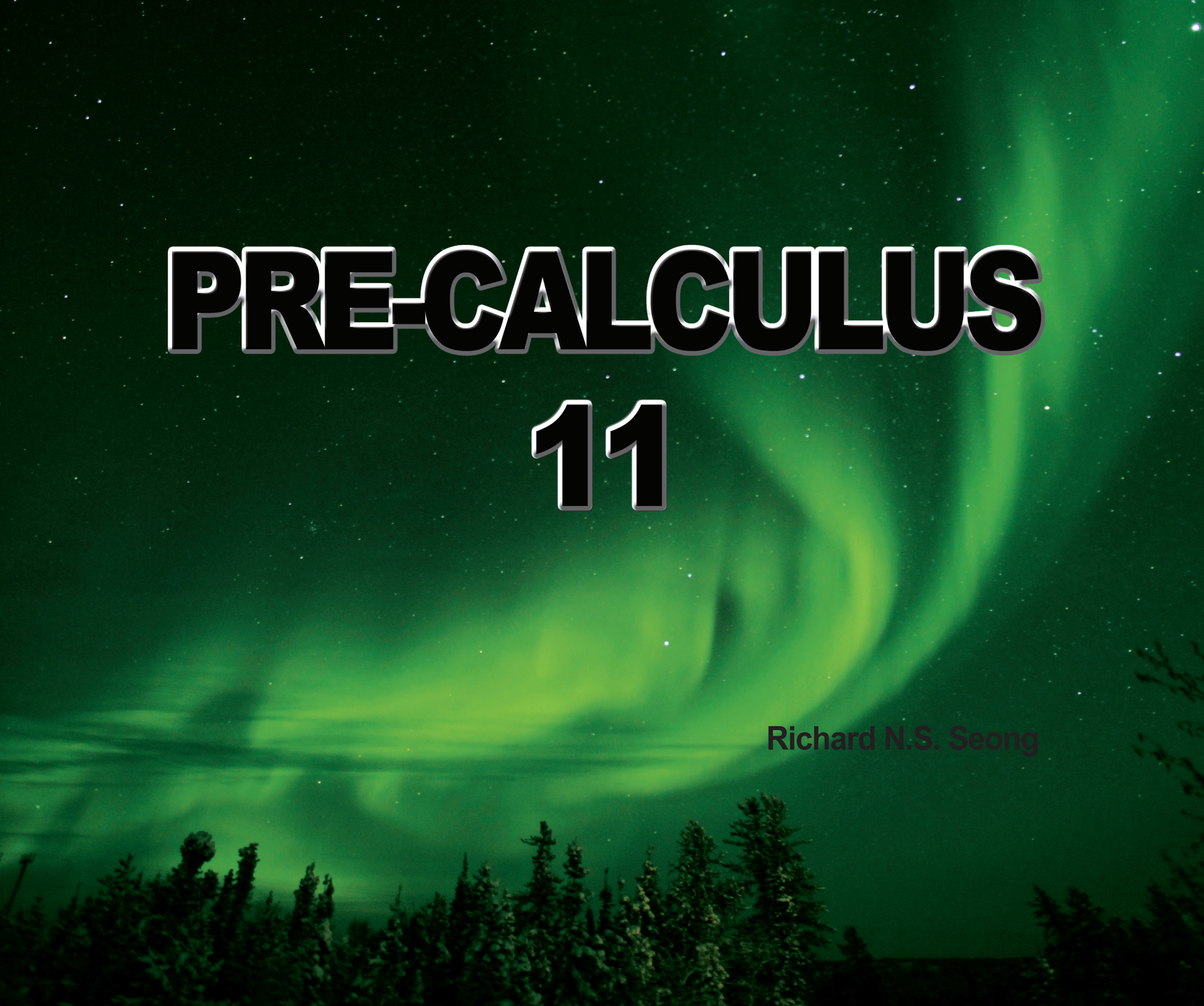
PRE-CALCULUS 11



Richard N.S. Seong

- Key points concisely summarized
- More than 450 complete guided examples
- 3000 carefully crafted questions with full solutions!
- 1400 graphs and diagrams to help explain and solve problems!

Infinite Challenge Publishing Ltd.

Table of Contents

Chapter 1. Polynomials

Review of Exponent Laws....7
Operations with Polynomials....9
Factoring Polynomials....18
Solving Quadratic Equations by Factoring....26

Chapter Review Exercises....31

Chapter 2. Radical Expressions and Equations

Review of the Real Number System....35
Roots and Radicals....38
Simplifying Radicals....44
Adding and Subtracting Radicals....49
Multiplying Radicals....53
Rationalizing Denominators....58
Solving Radical Equations....64

Chapter Review Exercises....71

Chapter 3. Rational Expressions, Equations, and Inequalities

Simplifying Rational Expressions....76
Multiplying and Dividing Rational Expressions....80
Adding and Subtracting Rational Expressions....86
Solving Rational Equations....95
Applications of Rational Equations....100
Solving Rational Inequalities Algebraically....105

Chapter Review Exercises....109

Chapter 4. Quadratic Equations and Inequalities

Solving Quadratic Equations by Factoring....114
Solving Quadratic Equations Using Square Root Property....121
Solving Quadratic Equations by Completing the Square....125
Solving Quadratic Equations Using Quadratic Formula....129
Solving Quadratic Equations Using a Graphing Calculator....134
Discriminant: $D = b^2 - 4ac$....137
Sum and Product of the Roots....141
Solving Quadratic Inequalities: Algebraic Method....147
Applications of Quadratic Equations / Inequalities....157

Chapter Review Exercises....167

Chapter 5. Quadratic Functions

Definition of Quadratic Function....173
Graphing $y = ax^2$....176
Graphing $y = ax^2 + q$....180
Graphing $y = a(x-p)^2$....186
Graphing $y = a(x-p)^2 + q$....189
Graphing Parabolas in factored form, $y = a(x-r_1)(x-r_2)$....198
Graphing $y = ax^2 + bx + c$....202

Solving Quadratic Inequalities: Graphical Method....215
Applications of Quadratic Functions....220

Chapter Review Exercises....229

Chapter 6. Systems of Equations and Inequalities

Union and Intersection: OR & AND....236
Solving Systems of Linear Equations in Two Variables....243
Solving Systems of Equations Involving Quadratic Equations....251
Solving Systems of Inequalities in Two Variables....263

Chapter Review Exercises....272

Chapter 7. Trigonometry

Angular Measure....277
Trigonometric Ratios....280
Special Angles and Solving Trigonometric Equations....284
Solving Trigonometric Equations Using a Calculator: Algebraic Method....294
Law of Sines....296
Law of Cosines....300
Ambiguous Case (SSA)....307
Area of a Triangle....314

Chapter Review Exercises....318

Chapter 8. Absolute Value, Reciprocal, and Exponential Functions

Absolute Value....323
Graphing Absolute Value Functions....327
Solving Absolute Value Equations....335
Solving Absolute Value Inequalities....341
Graphing Reciprocal Functions....345
Graphing Exponential Functions, $y=a(b)^x$....354

Chapter Review Exercises....359

Chapter 9. Financial Literacy

Simple Interest....364
Compound Interest....368
Solving Compound Interest Problems Using a Graphing Calculator (TVM Solver)....375
Investments with Annuities....380
Investment Portfolios....385
Loans....389
Credit Cards....392
Mortgages....398
Buying, Renting, or Leasing....404
Modeling Financial Growth/Decay with Exponential Functions....410

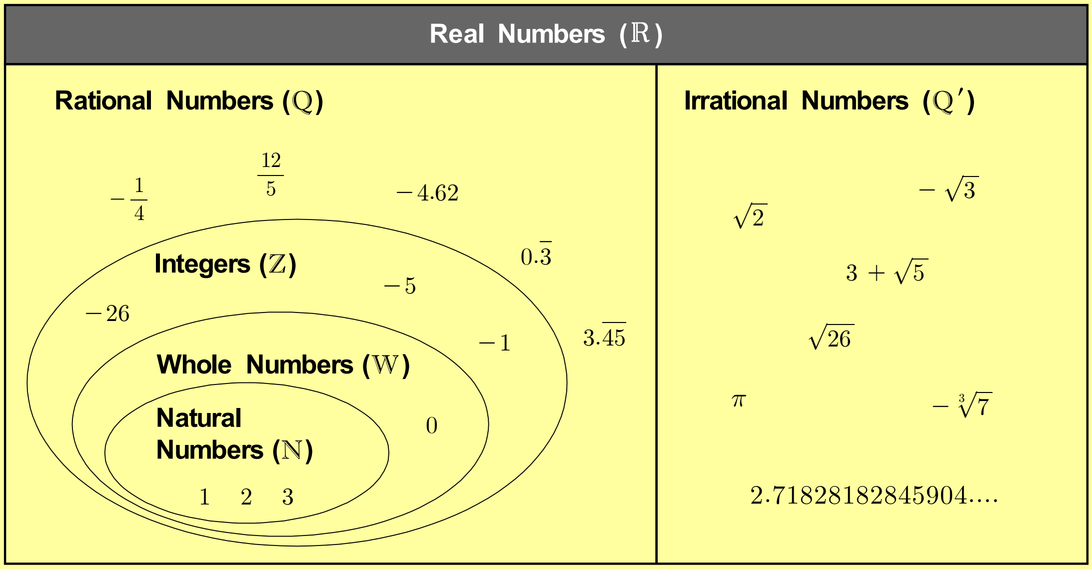
Chapter Review Exercises....413

***Solutions**....417 – 521

2. Radical Expressions and Equations

★ Review of the Real Number System

◆ The Real Number System



Natural Numbers ($\mathbb{N}$) : $\{1,\ 2,\ 3,\ 4,\ ......\}$

Whole Numbers ($\mathbb{W}$) : $\{0,\ 1,\ 2,\ 3,\ 4,\ ......\}$

Integers ($\mathbb{Z}$) : $\{0,\ \pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ ......\}$

Rational Numbers ($\mathbb{Q}$) : Any real numbers that can be written in the form $\frac{p}{q}$.
(where $p \in \mathbb{Z}$, $q \in \mathbb{Z}$ and $q \neq 0$)

(e.g.) $-4.62 = -\frac{231}{50}$, $0.\overline{3} = \frac{1}{3}$, $3.\overline{45} = \frac{38}{11}$

Irrational Numbers ($\mathbb{Q}'$) : Any real numbers that cannot be written in the form $\frac{p}{q}$.
Irrational numbers are non-repeating and non-terminating decimals.

(e.g.) $\sqrt{2} = 1.414213562....$, $\sqrt{3} = 1.732050807....$

(Example 1) Name all the sets of numbers to which each number belongs.
Let $\mathbb{N}$=natural numbers, $\mathbb{W}$=whole numbers, $\mathbb{Z}$=integers, $\mathbb{Q}$=rational numbers, $\mathbb{Q}'$=irrational numbers, and $\mathbb{R}$=real numbers.

① 6

$\mathbb{N}$, $\mathbb{W}$, $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$

② $-\sqrt[3]{27}$

$\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$ (because $-\sqrt[3]{27} = -3$.)

③ $0.\overline{4}$

$\mathbb{Q}$ and $\mathbb{R}$ (because $0.\overline{4} = \frac{4}{9}$.)

④ $\sqrt{15}$

$\mathbb{Q}'$ and $\mathbb{R}$

(Since 15 is not a perfect square, $\sqrt{15}$ is a non-repeating and non-terminating decimal.)

[EX6] Determine whether each of the following statements is true or false. If false, explain why.

① $\sqrt{4} = \pm 2$

② $\sqrt{-4} = -2$

③ If $x^2 = 4$, then $x = \pm 2$.

④ $\sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2$

⑤ $\sqrt{16+25} = 4+5$

⑥ $\sqrt[4]{(-2)^4} = -2$

⑦ $\sqrt{x^2} = x$, for all real numbers.

⑧ $(\sqrt{x})^2 = x$, for all real numbers.

⑨ $(\sqrt[3]{-2})^3 - \sqrt{(-2)^2} = -2 - |-2|$

⑩ $\sqrt{(x+1)^2} = x+1$, for all real numbers.

⑪ $\sqrt[3]{(x+1)^3} = x+1$, for all real numbers.

⑫ The domain of $(\sqrt{x+1})^2$ is $x \geq -1$.

⑬ $\sqrt[3]{-8^2} - (\sqrt[3]{-8})^2 = -4-4$

⑭ If $(x+2)^4 = 16$, then $x+2 = \sqrt[4]{16}$.

⑮ $\sqrt[3]{-x} = -\sqrt[3]{x}$, for all real numbers.

⑯ $(\sqrt{x+3})^2 = x+3$, for all real numbers.

[EX2] Simplify each rational expression. Identify all non-permissible values.

① $\dfrac{a^2-5a}{4a+4}\cdot\dfrac{5a+5}{a}$ ② $(m-3n)^2\cdot\dfrac{mn}{3n-m}$ ③ $(9x^2-y^2)\cdot\dfrac{y}{y+3x}$

④ $\dfrac{x^2-3x-4}{x^2-1}\times\dfrac{2x+2}{x^2-7x+12}$ ⑤ $\dfrac{x^2+7x+6}{-x^2-2x+24}\times\dfrac{x^2-16}{x^2+8x}$

⑥ $\dfrac{2a^2+3a-5}{4a^2+8a-5}\times\dfrac{4a^2-4a+1}{2a^2-3a+1}$ ⑦ $\dfrac{12m^2-23m+10}{3m^2-5m+2}\cdot\dfrac{m^2-3m+2}{20m-16m^2}$

⑧ $\dfrac{6x^2-5xy-4y^2}{4x^2-4xy-3y^2}\cdot\dfrac{4x^2-12xy+9y^2}{16y^2-9x^2}$

Challenge

⑨ $\dfrac{x^5-5x^3+4x}{x^2-1}\cdot\dfrac{9-4x^2}{2x^2+x-6}$

(Example 2) Richard and Alice ride the bicycle for 9 km from school to get home. One day, Richard rode 3 km/h faster than Alice and arrived home half an hour earlier, how fast did each person ride?

Let x be Alice's speed in km/h.

	Speed (km/h)	Distance (km)	Time (hr)
Alice	x	9	$\frac{9}{x}$
Richard	$x+3$	9	$\frac{9}{x+3}$
Difference	-	-	$\frac{1}{2}$

$$\frac{9}{x} - \frac{9}{x+3} = \frac{1}{2}$$

$$2x(x+3)\cdot\left(\frac{9}{x} - \frac{9}{x+3}\right) = \frac{1}{2}\cdot 2x(x+3)$$

$$18(x+3) - 18x = x(x+3)$$

$$18x + 54 - 18x = x^2 + 3x$$

$$54 = x^2 + 3x$$

$$x^2 + 3x - 54 = 0$$

$$(x-6)(x+9) = 0$$

$$x = 6 \ \ or \ \ \cancel{-9} \quad (reject \ \because x > 0)$$

$\therefore$ Alice's speed $= 6 \ km/h$

Richard's speed $= 6 + 3 = 9 \, km/h$

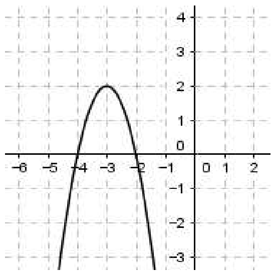
[EX3] To travel 200 km, it takes 30 minutes more to drive than to take the train. If the average speed of the train is 20 km/h faster than that of the car, what are the average speeds of the car and the train?

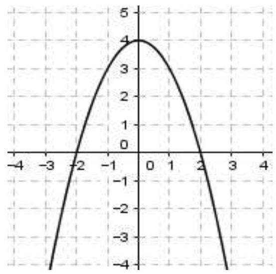
[EX4] Yas can run 2 m/s faster than Nancy. If Yas took 20 seconds more to run 1200 meters than Nancy did to run 800 meters, how fast were they running?

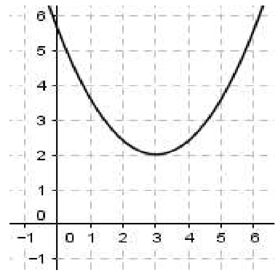
[EX4] Match each quadratic function with its graph. Do not use a graphing calculator.

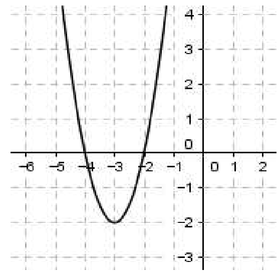
① $y=(x-2)^2$ ② $y=-\frac{3}{2}x^2+4$ ③ $y=-\frac{3}{2}(x+4)^2$ ④ $y=4-x^2$

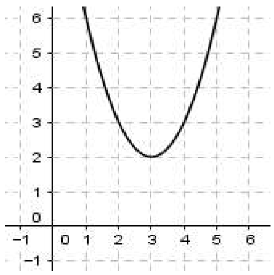
⑤ $y=(x-3)^2+2$ ⑥ $y=2(x+3)^2-2$ ⑦ $y=\frac{2}{5}(x-3)^2+2$ ⑧ $y=-2(x+3)^2+2$

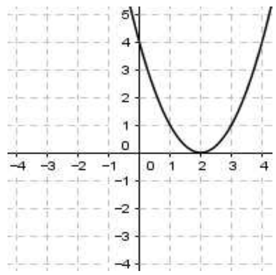
(a)


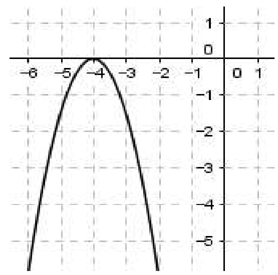
(b)


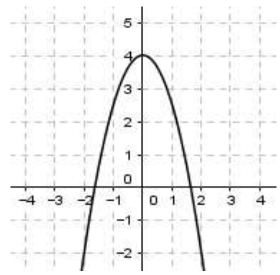
(c)


(d)


(e)


(f)


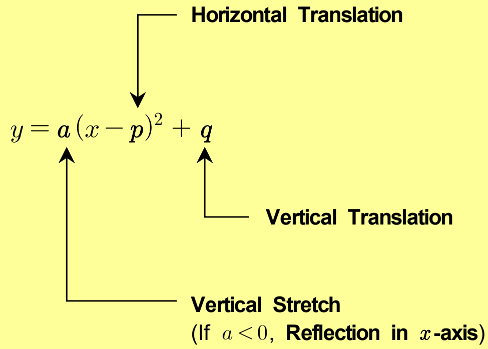
(g)


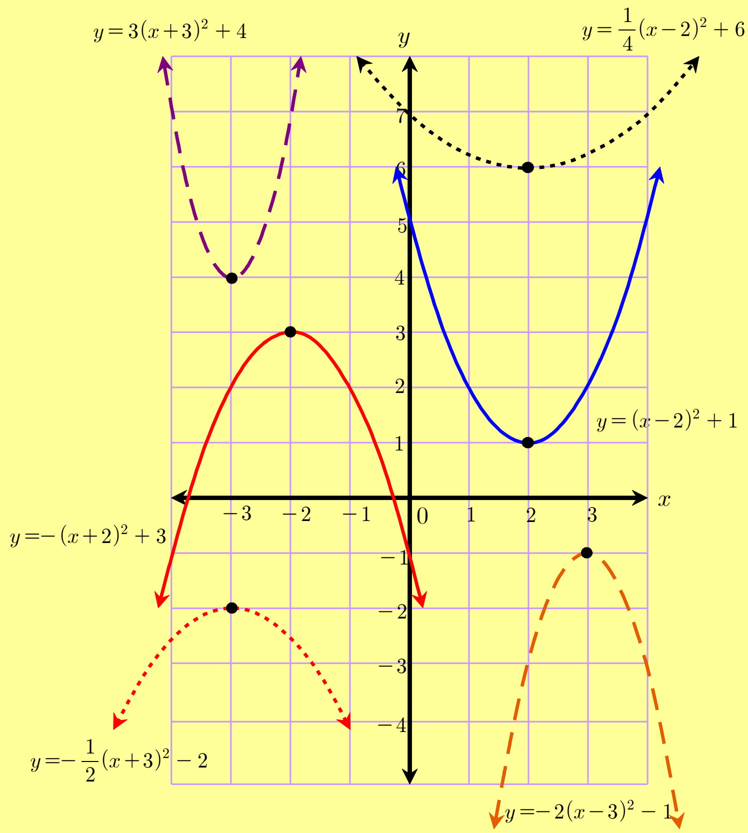
(h)


[EX5] Complete the table below.

	direction of opening	vertex	axis of symmetry	maximum value	minimum value	y-intercept	domain	range
① $y=(x-5)^2-12$	up	$(5,-12)$	$x=5$	none	-12	13	$x \in R$	$y \geq -12$
② $y=-4(x+3)^2$								
③ $y=3x^2-4$								
④ $y=-2(x+2)^2+3$								
⑤ $y=-(x-4)^2+11$								
⑥ $y=\frac{(x+1)^2}{5}+2$								
⑦ $y=-\frac{2}{3}x^2-9$								

★ Graphing $y = a(x-p)^2 + q$





◆ Summary

(1) The graph of $y = a(x-p)^2 + q$ is obtained by **translating** $y = ax^2$ **horizontally by** p **units** and **vertically** q **units.**

- If $p > 0$, **move** p units **to the right** ; If $p < 0$, **move** $|p|$ units **to the left.**
- If $q > 0$, **move up** q units; If $q < 0$, **move down** $|q|$ units.

(2) The parabola $y = a(x-p)^2 + q$ has its **vertex at** (p, q) and its **axis of symmetry on the line** $x = p$.

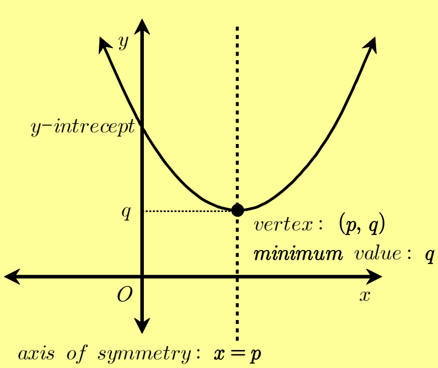
(3) If $a > 0$, the parabola opens upward and has the **minimum** value of q at the vertex.

Domain: $x \in R$, Range: $y \geq q$

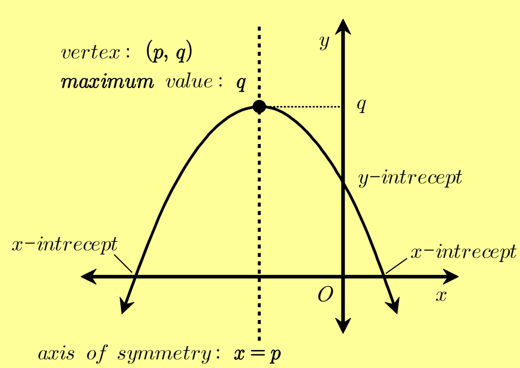
(4) If $a < 0$, the parabola opens downward and has the **maximum** value of q at the vertex.

Domain: $x \in R$, Range: $y \leq q$

$y = a(x-p)^2 + q$ *with* $a > 0$



$y = a(x-p)^2 + q$ *with* $a < 0$



[EX1] Solve each inequality **graphically**. Do not use a calculator.

① $x^2+5x>0$ ② $x^2-9<0$ ③ $x^2-5x>14$

④ $x^2 \le 6x-5$ ⑤ $6x-2x^2 \le 0$ ⑥ $-3x^2-x+10 \ge 0$

⑦ $-5t^2-4t+1 \ge 0$ ⑧ $6x(x-2)>x+5$

Challenge

[EX2] Find all values of x for which $f(x)=3x^2+5x$ lies above $g(x)=3x+8$.

[EX6] Without graphing, determine whether each system has one solution, no solutions, or infinitely many solutions.

① $\begin{cases} 2x - y = 0 \\ 6x - 3y = 2 \end{cases}$ ② $\begin{cases} 3x + 2y = 6 \\ 9x + 6y = 18 \end{cases}$ ③ $\begin{cases} y = \dfrac{2}{3}x - 3 \\ y = \dfrac{2}{3}x + 2 \end{cases}$

④ $\begin{cases} 3x - 4y = 6 \\ 4x - 3y = 6 \end{cases}$ ⑤ $\begin{cases} y = \dfrac{1}{2}x + 2 \\ x - 2y + 4 = 0 \end{cases}$ ⑥ $\begin{cases} x - 3y = 3 \\ -\dfrac{x}{3} + y = -1 \end{cases}$

[EX7] Determine whether each system is consistent or inconsistent.

① $\begin{cases} 2x + y = 4 \\ x - y = 4 \end{cases}$ ② $\begin{cases} 3x - 2y = 6 \\ 2y - 3x = 12 \end{cases}$ ③ $\begin{cases} y = -\dfrac{1}{3}x + 4 \\ 2x + 6y = 24 \end{cases}$

(Example 6) Consider the system of equations $\begin{cases} y = -2x + k \\ mx + 2y = 12 \end{cases}$.

Find the values of m and k such that the system has: (1) infinitely many solutions (2) no solutions.

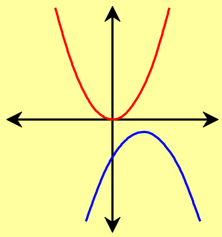
(1) Rewrite in standard form: $\begin{cases} 2x + y = k \\ mx + 2y = 12 \end{cases}$

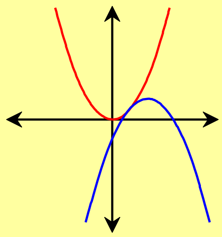
For the system to have infinitely many solutions, $\dfrac{2}{m} = \dfrac{1}{2} = \dfrac{k}{12} \Rightarrow \therefore\ m = 4,\ k = 6$

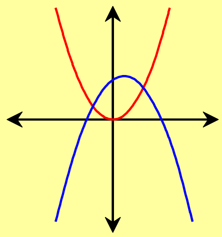
(2) For the system to have no solutions, $\dfrac{2}{m} = \dfrac{1}{2} \neq \dfrac{k}{12} \Rightarrow \therefore\ m = 4,\ k \neq 6$

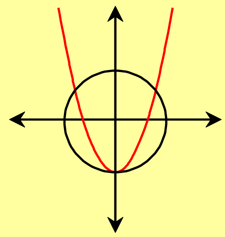
◆ Quadratic-Quadratic Systems

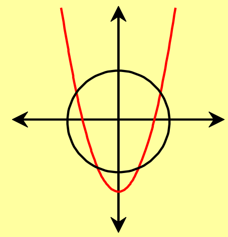
A quadratic-quadratic system of equations in two variables consists of two quadratic equations with the form, $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. A solution of a quadratic-quadratic system is an ordered pair (x, y) that satisfies both equations in the system.
There are six possible solutions when solving a quadratic-quadratic system.

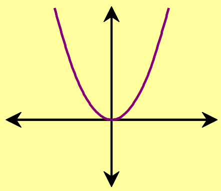

no solution


1 solution


2 solutions


3 solutions


4 solutions

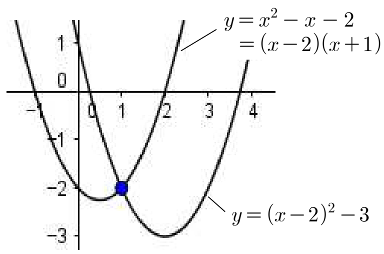

infinitely many solutions (coincident graphs)

◆ Solving Quadratic-Quadratic Systems Algebraically

Step 1. Solve one of the equations for one variable.
Step 2. Substitute the expression from step 1 into the other equation, and solve for the other variable.
Step 3. Substitute the solution into either original equation, then solve for the remaining variable.
Step 4. Write the solution as an ordered pair.

(Example 7) Graph each system and find all points of intersection algebraically.

① $\begin{cases} y = x^2 - x - 2 & (1) \\ y = (x-2)^2 - 3 & (2) \end{cases}$

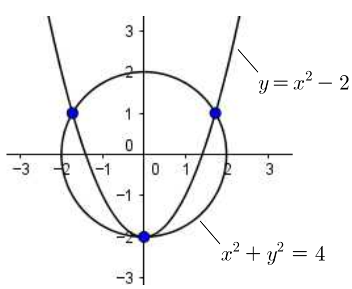


Substitute $x^2 - x - 2$ for y in equation (2), and solve for x.

$$x^2 - x - 2 = (x-2)^2 - 3$$
$$\cancel{x^2} - x - 2 = \cancel{x^2} - 4x + 4 - 3$$
$$3x = 3$$
$$\therefore\ x = 1$$

When $x = 1$, $y = (1)^2 - (1) - 2 = -2$. $\therefore$ The solution is $(1, -2)$.

② $\begin{cases} x^2 + y^2 = 4 & (1) \\ y = x^2 - 2 & (2) \end{cases}$



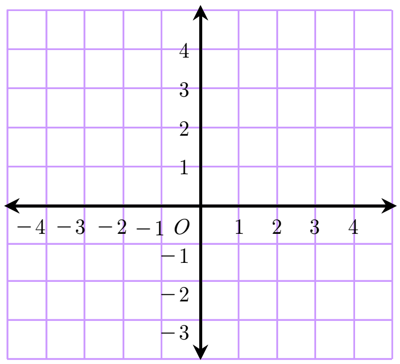
Solve (2) for x^2: $x^2 = y + 2$

Substitute $y + 2$ for x^2 in equation (1), and solve for y.

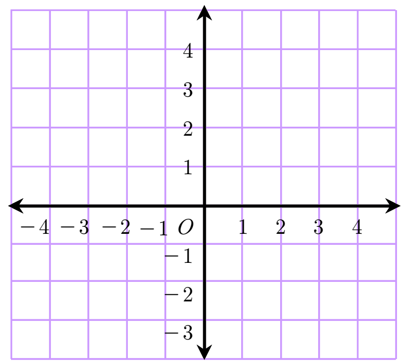
$$(y+2) + y^2 = 4$$
$$y^2 + y - 2 = 0$$
$$(y+2)(y-1) = 0$$
$$\therefore\ y = -2,\ 1$$

When $y = -2$, $x^2 = -2 + 2 = 0$.
$x = 0$
When $y = 1$, $x^2 = 1 + 2 = 3$.
$x = \pm\sqrt{3}$ $\therefore$ The solution is $(0, -2)$ *or* $(\pm\sqrt{3}, 1)$.

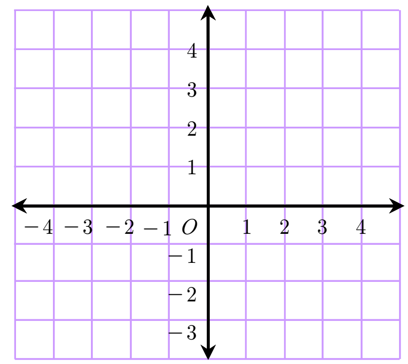
④ $x^2 + 2y \le 0$



⑤ $y < x^2 - 6x + 8$

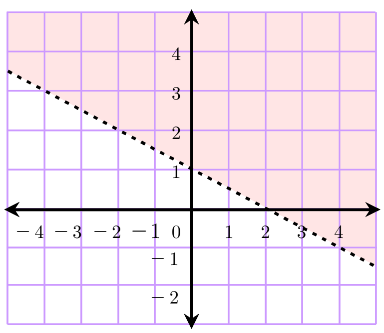


⑥ $y \ge -x^2 + 4x$



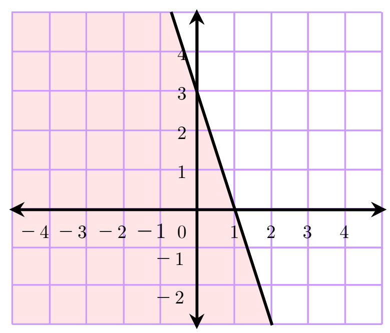
[EX4] Write an inequality that represents each graph.

①



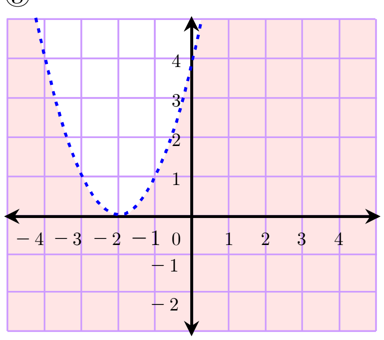
____

②



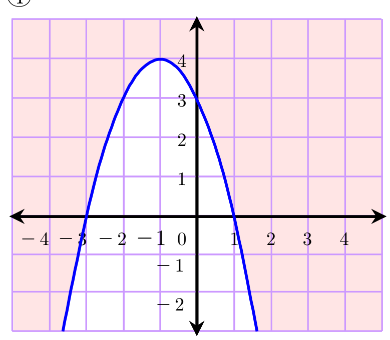
____

③



____

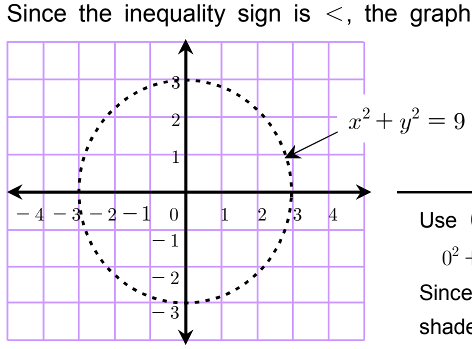
④



____

(Example 2) Graph the inequality: $x^2 + y^2 < 9$

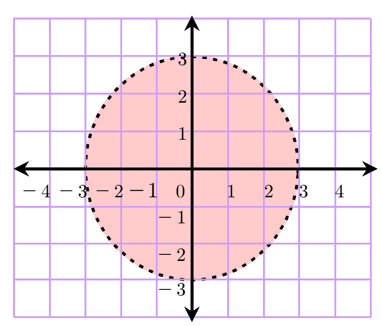
The graph of $x^2 + y^2 = 9$ is a circle with a radius of 3, centered at the origin .
Since the inequality sign is $<$, the graph is drawn as a dashed circle.



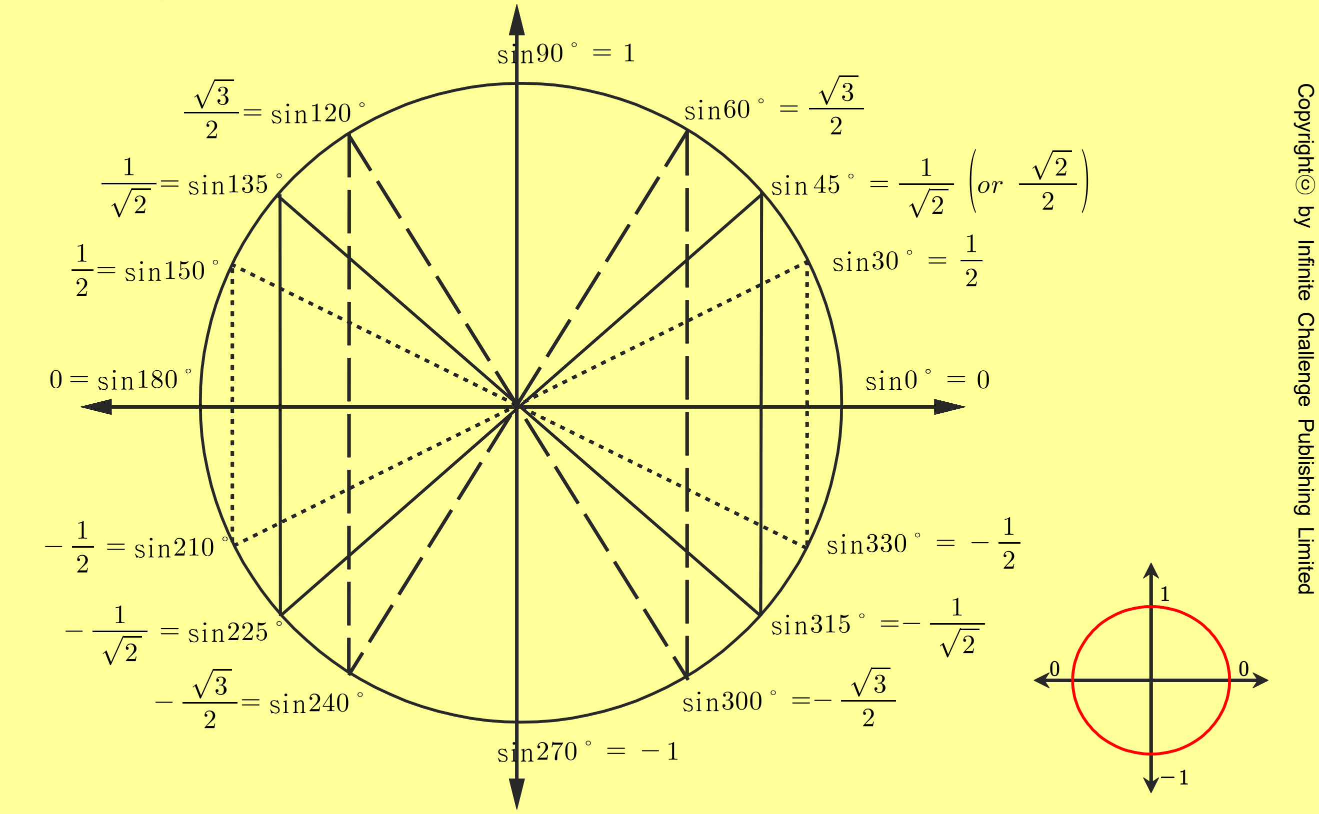
Use $(0, 0)$ as a test point.

$0^2 + 0^2 < 9$

Since the origin satisfies the inequality, shade the region containing the origin.

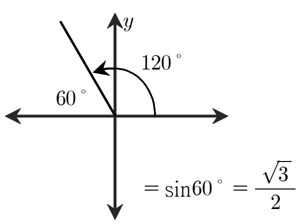


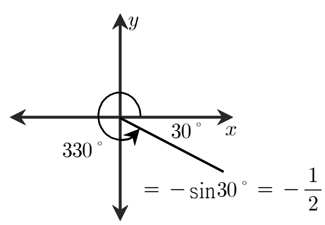
◆ $\sin\theta = \dfrac{y}{r}$

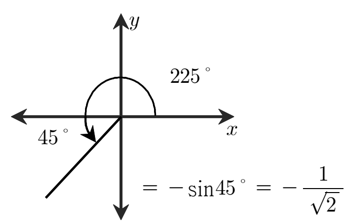


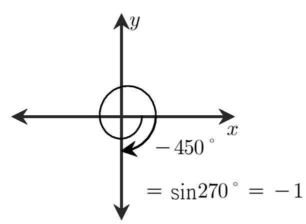
(Example 1) Evaluate.

① $\sin 120^\circ$ ② $\sin 330^\circ$ ③ $\sin 225^\circ$ ④ $\sin(-450^\circ)$









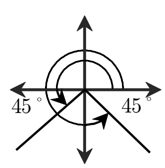
(Example 2) Solve for θ $(0^\circ \le \theta < 360^\circ)$.

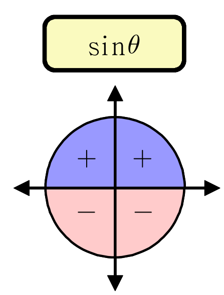
$\sin\theta = -\dfrac{1}{\sqrt{2}}$

Step 1. Find the reference angle.

$ref\angle = 45^\circ$

Step 2. Draw the terminal arms in standard position. Since the sine value is negative, the terminal arms must be in the 3rd and 4th quadrant.





Step 3. Find the rotation angles in the given domain.

$\therefore\ \theta = 180 + 45^\circ,\ 360 - 45^\circ$

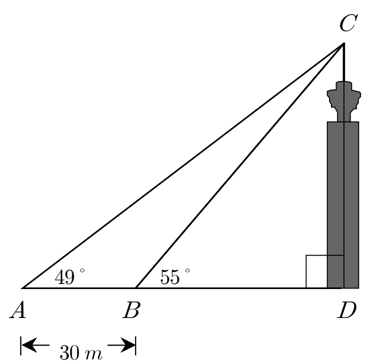
$= 225^\circ,\ 315^\circ$

③ In $\triangle XYZ$, $\angle X = 58^\circ$, $\angle Y = 85^\circ$, and $x = 13\ cm$.

④ In $\triangle PQR$, $\angle Q = 72^\circ$, $\angle R = 46^\circ$, and $p = 8\ m$.

⑤ In $\triangle ABC$, $\angle A = 110^\circ$, $\angle B = 25^\circ$, and $c = 9\ ft$.

(Example 2) An engineer wishes to calculate the height of the Harbour Centre in Vancouver. The angle of elevation to the top of the building is 49° from point A and 55° from point B, as shown below. If A and B are $30\ m$ apart, find the height of the Harbour Centre.



Step 1. In $\triangle ABC$, use the Exterior Angle Theorem to find $\angle ACB$.

$$\angle ACB = 55^\circ - 49^\circ = 6^\circ$$

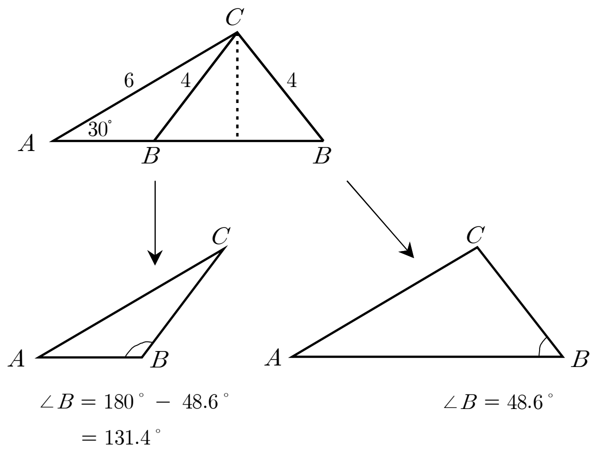
Step 2. In $\triangle ABC$, use the law of sines to find BC.

$$\frac{30}{\sin 6^\circ} = \frac{BC}{\sin 49^\circ} \quad \Rightarrow \quad BC = \frac{30 \sin 49^\circ}{\sin 6^\circ} \approx 216.604$$

Step 3. In $\triangle BCD$, use sine ratio to find CD.

$$\sin 55^\circ = \frac{CD}{216.604} \quad \Rightarrow \quad CD = 216.604 \sin 55^\circ \approx 177.4 m$$

③ $\angle A = 30^\circ$, $a = 4$, and $b = 6$.



$h = b\sin A$

$h = 6\sin 30^\circ = 3 < 4 < 6$

Since $h < a < b$, **two triangles** can be drawn.

Use the law of sines.

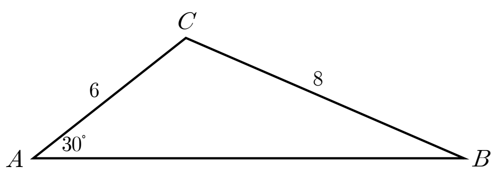
$$\frac{6}{\sin B} = \frac{4}{\sin 30^\circ}$$

$$\sin B = \frac{6\sin 30^\circ}{4} = 0.75 \qquad \Rightarrow \qquad \therefore\ \angle B = 48.6^\circ$$

If $\angle B$ is obtuse, $\angle B = 180^\circ - 48.6^\circ = 131.4^\circ$.

$\therefore\ \angle B = 48.6^\circ\ or\ \ 131.4^\circ$

④ $\angle A = 30^\circ$, $a = 8$, and $b = 6$.



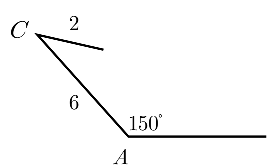
Since $a > b$, only **one triangle** can be drawn.

Use the law of sines.

$$\frac{6}{\sin B} = \frac{8}{\sin 30^\circ}$$

$$\sin B = \frac{6\sin 30^\circ}{8} = 0.375 \qquad \Rightarrow \qquad \therefore\ \angle B = 22.0^\circ$$

⑤ $\angle A = 150^\circ$, $a = 2$, and $b = 6$.



(Method 1) Since $\angle A$ is obtuse and $a < b$, **no triangle** can be drawn.

$\therefore$ There is no solution for $\angle B$.

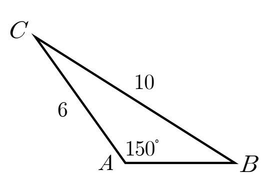
(Method 2) Use the law of sines.

$$\frac{6}{\sin B} = \frac{2}{\sin 150^\circ}$$

$$\sin B = \frac{6\sin 150^\circ}{2} = 1.5$$

Since $\sin B$ cannot be greater than 1, there is no solution for $\angle B$.

⑥ $\angle A = 150^\circ$, $a = 8$, and $b = 6$.



Since $\angle A$ is obtuse and $a > b$, only **one triangle** can be drawn.

Use the law of sines.

$$\frac{6}{\sin B} = \frac{10}{\sin 150^\circ}$$

$$\sin B = \frac{6\sin 150^\circ}{10} = 0.3$$

$$\therefore\ \angle B = 17.5^\circ$$

Chapter Review Exercises

Do not use a calculator unless stated otherwise.

1. Find all angles θ that satisfy the given conditions.

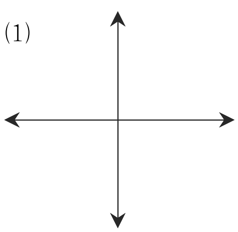
① $0^\circ \leq \theta \leq 720^\circ$ and θ is coterminal with -150° ② $0^\circ \leq \theta \leq 720^\circ$ and θ is coterminal with 1200°

③ $-720^\circ \leq \theta \leq 0^\circ$ and θ is coterminal with -1000°

2. (1) Draw the following rotation angles in standard position.
(2) Find the principal angle.
(3) Find the reference angle.
(4) Find the <u>next</u> positive coterminal angle and the <u>first</u> negative coterminal angle.
(5) Write an expression using the principal angle to represent all coterminal angles.

① $\theta = 500^\circ$

(1)



(2)____________________

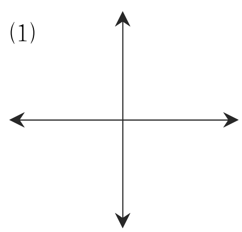
(3)____________________

(4)____________________

(5)____________________

② $\theta = -1500^\circ$

(1)



(2)____________________

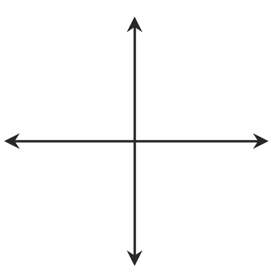
(3)____________________

(4)____________________

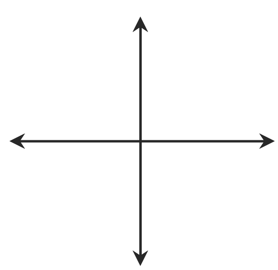
(5)____________________

3. Find the indicated trigonometric ratio with the given information.

① The point $P(4, -3)$ lies on the terminal arm of θ in standard position. Find the value of $\sin\theta$.



② The point $P(-2, 0)$ lies on the terminal arm of θ in standard position. Find the value of $\cos\theta$.

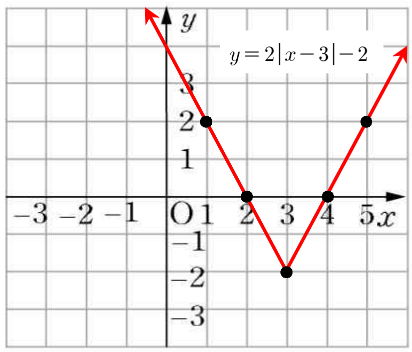


(Example 2) Consider the absolute value function $y=|6-2x|-2$.
(1) Write the absolute function in the form $y=a|x-p|+q$. (2) Sketch the graph of the function and find all intercepts. (3) Rewrite as a piecewise function.

(1) $y=|6-2x|-2 \Leftrightarrow y=|2x-6|-2$ (since $|a-b|=|b-a|$.)
$\Leftrightarrow y=|2||x-3|-2$ (since $|ab|=|a||b|$.)
$\Leftrightarrow y=2|x-3|-2$

(2) Vertex: $(3,-2)$

x	y
1	2
2	0
3	-2
4	0
5	2



y-intercept $=4$; x-intercept $=2,\ 4$

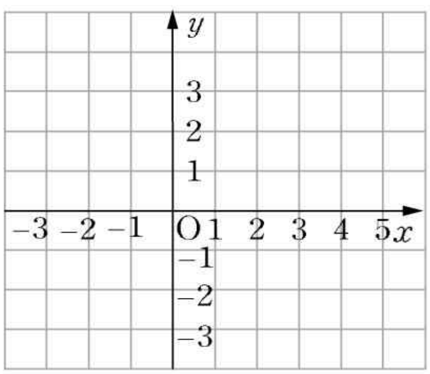
(3)

$$y=\begin{cases} 2(x-3)-2, & if\ x-3\geq 0 \\ -2(x-3)-2, & if\ x-3<0 \end{cases}$$

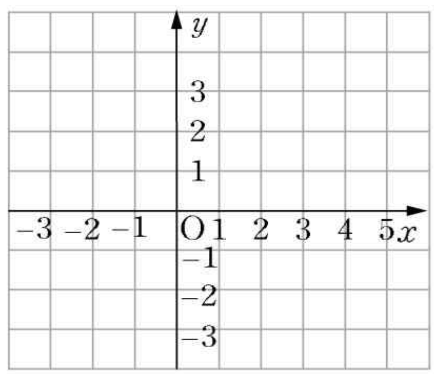
$$\therefore\ y=\begin{cases} 2x-8, & if\ x\geq 3 \\ -2x+4, & if\ x<3 \end{cases}$$

[EX2] (1) Write each function in the form $y=a|x-p|+q$. (2) Sketch the graph of the function and find all intercepts. (3) Rewrite as a piecewise function.

① $y=-\frac{1}{2}|2x+2|+2$



② $y=|6-3x|-3$



⑬ $\left|\dfrac{n-1}{4}\right|-2\geq 1$

⑭ $-2\left|\dfrac{1}{3}p-3\right|+5\geq -3$

◆ **Solving $|f(x)|>g(x)$, $|f(x)|<g(x)$, $|f(x)|\geq g(x)$, and $|f(x)|\leq g(x)$ Algebraically**

Step 1. Write the inequality as an absolute equation, then solve the equation.

Step 2. **Check each solution** in the original equation and reject any extraneous roots.
The real roots to the equation are **boundary points**.

Step 3. Locate the boundary points on a number line, then divide the number line into intervals.
- If the inequality is $>$ or $<$, mark the boundary points with **open** circles.
- If the inequality is $\geq$ or $\leq$, mark the boundary points with **closed** circles.

Step 4. Test an x-value in each interval and find the solution set.

(Example 2) Solve $|2x-3|\leq x$.

(Method 1) Solving Algebraically

Step 1. Let $|2x-3|=x$.

$2x-3=x$ or $2x-3=-x$

$2x-3=x$	$2x-3=-x$
$2x-x=3$	$2x+x=3$
$x=3$	$3x=3$
	$x=1$

Step 2. Check: If $x=3$, $|2(3)-3|=3$
$|3|=3$ (True)

If $x=1$, $|2(1)-3|=1$
$|-1|=1$ (True)

$\therefore\ x=1,\ 3$ **(boundary points)**

Step 3.

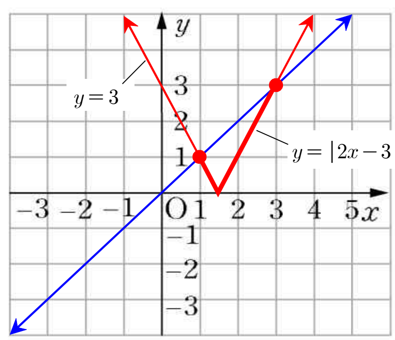
Step 4.

Test 0	Test 2	Test 5
$\lvert 2(0)-3\rvert\leq 0$	$\lvert 2(2)-3\rvert\leq 2$	$\lvert 2(5)-3\rvert\leq 5$
$\lvert -3\rvert\leq 0$	$\lvert 1\rvert\leq 2$	$\lvert 7\rvert\leq 5$
False	**True**	**False**

$\therefore\ 1\leq x\leq 3$

(Method 2) Solving Graphically

Graph the functions $y=|2x-3|$ and $y=x$ on the same grid.



The graphs intersect at $x=1,\ 3$.

The graph of $y=|2x-3|$ lies **on or below** the graph of $y=x$ when $1\leq x\leq 3$.

$\therefore\ 1\leq x\leq 3$

★ Investments with Annuities

◆ An **annuity** is **a series of periodic payments** of equal amount.

- **Ordinary annuity**: payments made at the end of each period.
  (e.g.) saving deposits, loan payments, house mortgage, etc.
- **Annuity due**: payments made at the beginning of each period.
  (e.g.) life insurance, lease payment, house rent, etc.

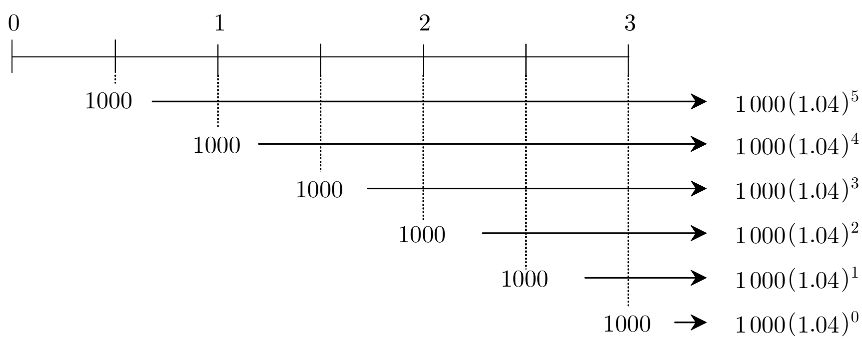
◆ **Future Value of an Ordinary Annuity**

$$P(1+i)^0 + P(1+i)^1 + P(1+i)^2 + \cdots + P(1+i)^{n-1} = \frac{P[(1+i)^n - 1]}{i}$$

where P is the regular payment, i is the interest rate per period, and n is the number of payments.

(Example 1) Richard deposits $1 000 into a savings account at the end of every 6 months. If the account pays 8% p.a. compounded semi-annually, find the future value after 3 years.

(Method 1) **Applying Math** Interest rate per 6 months $= \frac{8\%}{2} = 4\%$

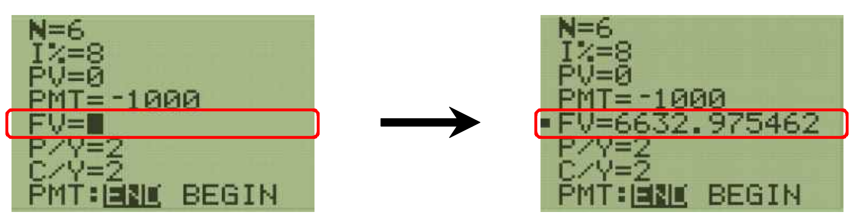


$$1000(1.04)^0 + 1000(1.04)^1 + 1000(1.04)^2 + 1000(1.04)^3 + 1000(1.04)^4 + 1000(1.04)^5 = \frac{1000(1.04^6 - 1)}{0.04}$$
$$= \$6\,632.98$$

$\therefore\ \$6\,632.98$

(Method 2) **TVM Solver**

N $= 3 \times 2 = 6$
I% $= 8\%$
PV $= 0$
PMT $= -1\,000$
(negative sign b/c money is paid)
FV $= ?$
P/Y $= 2$
C/Y $= 2$
PMT: END



Enter the values into the calculator except the FV value.
Move the cursor to **FV=**, then press **ALPHA** **ENTER**.

$\therefore\ \$6\,632.98$

(2) Calculate the accumulated value of her portfolio and the rate of return.

Mutual fund $= \$10\,000(1.12)(0.94)(1.11) = \underline{\$11\,686.08}$

Stock $= \$48.50 \times 200 = \underline{\$9\,700}$

Savings account

N $= 3 \times 12 = 36$

I% $= 3.75\%$

PV $= 0$

PMT $= -500$

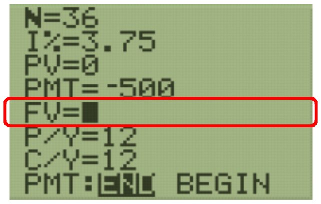
(negative sign b/c money is paid)

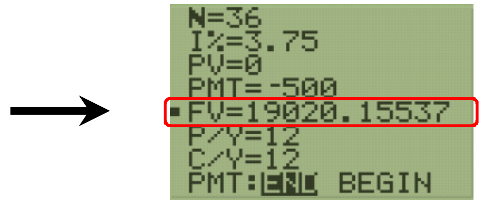
FV $= ?$

P/Y $= 12$

C/Y $= 12$

PMT: END





Enter the values into the calculator except the FV value.
Move the cursor to **FV=**, then press **ALPHA** **ENTER**.

$\therefore\ \underline{\$19\,020.16}$

$\therefore$ The accumulated value of the portfolio after 3 years $= \$11\,686.08 + \$9\,700 + \$19\,020.16$
$= \underline{\$40\,406.24}$

$\therefore$ The rate of return $= \dfrac{\$40\,406.24 - \$34\,850}{\$34\,850} \approx 0.159 \quad (or\ 15.9\%)$

[EX1] Sarah has made the following investment portfolio for the past 3 years:

- $5 700 was invested in a **government bond**. The bond pays interest at 5% compounded semi-annually.
- $7 000 was invested in a **mutual fund** that had annual returns of 2%, 10%, and 7% in each of the successive years.
- $250 was invested in a **savings account** at the end of every month for 3 years. The account pays interest at 2.75% compounded monthly.

(1) Calculate the total amount of money invested.

(2) Calculate the accumulated value of her portfolio and the rate of return. Round to the nearest tenth of a percent.

★ Loans

Money can be borrowed from financial institutions for many reasons: to buy a car, open a business, pay for education, remodel a house, etc.

When a loan is received from a financial institution, regular payments of principal and interest must be made over a specified period of time.

This process of paying off the loan with regular payments is called **amortization**.

(Example 1) Philip borrows $4 600 from a bank at 12% p.a compounded quarterly to buy a used car. He plans to repay the loan over 2 years.

① Calculate the payment he has to make at the end of every 3 months.

N $= 2 \times 4 = 8$

I% $= 12\%$

PV $= 4\,600$

(positive sign b/c money is received)

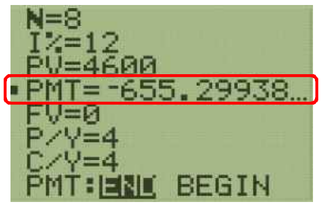
PMT $= ?$

FV $= 0$

P/Y $= 4$

C/Y $= 4$

PMT: END



Enter the values into the calculator except the PMT value.

Move the cursor to **PMT=**, then press **ALPHA** **ENTER**.

∴ $655.30

② Complete an amortization table for the loan.

Year	Opening balance (outstanding principal)	Interest charged	Regular payment	Closing balance (outstanding loan)
0.25	4 600.00	138.00	655.30	4 082.70
0.5	4 082.70	122.48	655.30	3 549.88
0.75	3 549.88	106.50	655.30	3 001.08
1	3 001.08	90.03	655.30	2 435.81
1.25	2 435.81	73.07	655.30	1 853.58
1.5	1 853.58	55.61	655.30	1 253.89
1.75	1 253.89	37.62	655.30	636.21
2	636.21	19.09	655.30	0.00

Note
The amount of outstanding principal decreases with each payment.
Therefore, the interest also decreases over time.

③ Calculate the total interest paid.

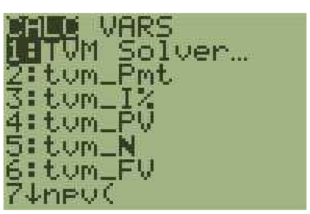
(Method 1) **Interest paid = Total amount paid − Principal**

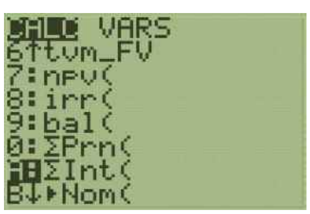
$= 655.30 \times 8 - 4600 = 642.40$ ∴ $642.40

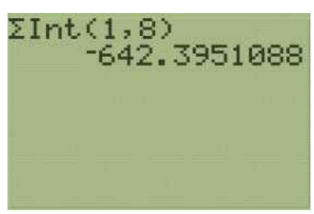
(Method 2) **TVM Solver**

⇒ Press **2nd** **MODE** **APPS** and select **1:Finance**, press **ENTER**.

Scroll down to **A:∑Int**, press **ENTER**. Type **1** **,** **8** **)**, then press **ENTER**.







∑Int $(1, N)$

∴ $642.40

③ Calculate the new balance.

④ Calculate the minimum payment.

(Example 2) Sam will purchase a used car for \$7 000 on credit. He can only afford monthly payments of \$300 and must choose one of the following two options:

- **Option A**: A dealership credit card with an interest rate of 18% compounded daily, with an immediate rebate of 3% off his first purchase
- **Option B**: A bank loan at 7.5% compounded monthly

① How long will it take to pay off the balance for the option A?

$N = ?$

$I\% = 18\%$

PV $= 7000 - 7000 \times 0.03$

PMT $= -300$

FV $= 0$

P/Y $= 12$

C/Y $= 365$

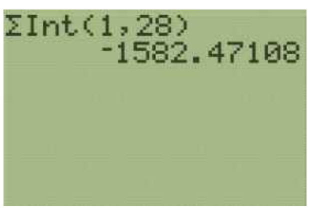
PMT: END

Enter the values into the calculator except the N value.

Move the cursor to **N=**, then press **ALPHA** **ENTER**.

Total number of interest payments(N) $= 27.908$ $\therefore$ 28 months

② What is the total interest paid for the option A?



$\therefore$ \$1 582.47

Refer to page 389

③ How long will it take to pay off the loan for the option B?

$N = ?$

$I\% = 7.5\%$

PV $= 7000$

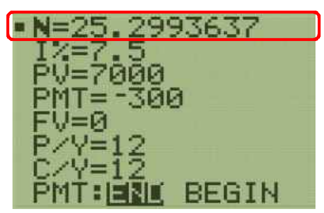
PMT $= -300$

FV $= 0$

P/Y $= 12$

C/Y $= 12$

PMT: END

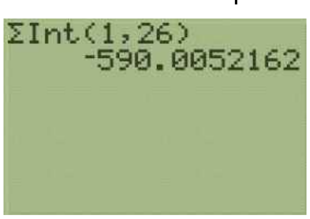


Enter the values into the calculator except the N value.

Move the cursor to **N=**, then press **ALPHA** **ENTER**.

Total number of interest payments(N) $= 25.299$ $\therefore$ 26 months

④ What is the total interest paid for the option B?



$\therefore$ \$590.01

[EX5] Olivia and Tyler are searching for a place to live. They have the following two options:

- **Option A**: **Rent** a condominium for $2 000 a month plus insurance for $250 and utilities for $2 300 each year.
- **Option B**: **Buy** a similar condominium for $475 000 with a 10% down payment and receive a 25-year mortgage for the balance at a fixed rate of 4.5% compounded semi-annually. The annual property tax is $1 750, but the income tax benefit is $890. Annual apartment maintenance costs will be $5 350.

① Calculate the average monthly cost for option A.

② Calculate the monthly mortgage payment in option B.

③ Calculate the average monthly cost for option B.

④ Calculate the total principal repaid for the first 3 years in option B.

⑤ Assuming that none of the costs change for 3 years, which option will cost less money? By how much?

[EX6] Nick has saved $50 000 and is searching for a place to live. He has the following two options:

- **Option A**: **Rent** an apartment for $1 600 a month plus insurance for $200 and utilities for $1 900 each year. **Invest** $50 000 in a fund, earning 7.5% compounded monthly, for 3 years.
- **Option B**: **Purchase** an apartment for $375 000 with $50 000 down payment and receive a 20-year mortgage for the balance at a fixed rate of 4.25% compounded semi-annually. The annual property tax is $1 350, but the income tax benefit is $640. Annual apartment maintenance costs will be $4 120.

SOLUTIONS

④ NPV: $t \neq 0,\ \frac{1}{2}$

$(t+2)(2t-1) = 3t$

$2t^2 + 3t - 2 = 3t$

$2t^2 - 2 = 0$

$t^2 = 1$

$t = \pm 1$

⑤ NPV: $n \neq 0,\ 3$

$8n = (n+3)(n-3)$

$8n = n^2 - 9$

$n^2 - 8n - 9 = 0$

$(n-9)(n+1) = 0$

$n = 9,\ -1$

⑥ NPV: $n \neq \frac{2}{3}$

$-4 = (p-3)(3p-2)$

$-4 = 3p^2 - 11p + 6$

$3p^2 - 11p + 10 = 0$

$(3p-5)(p-2) = 0$

$p = \frac{5}{3},\ 2$

⑦ NPV: $n \neq 1,\ 2$

$(x+2)(x-1) = (x-2)(x+3)$

$x^2 + x - 2 = x^2 + x - 6$

$-2 = -8$

$\therefore$ *No solution*

[EX2] Multiply both sides by LCD.

① NPV: $t \neq 0$, LCD: $2t$

$3 = 4t + 2(t+1)$

$3 = 4t + 2t + 2$

$1 = 6t$

$t = \frac{1}{6}$

② NPV: $a \neq 0$, LCD: $6a$

$9 - 2(a+1) = 12a$

$9 - 2a - 2 = 12a$

$7 = 14a$

$a = \frac{1}{2}$

③ NPV: $x \neq 0$, LCD: x

$x^2 + 6 = 5x$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x = 2,\ 3$

④ NPV: $x \neq 0$, LCD: x

$1 - x + 2x = 1$

$1 + x = 1$

$x = 0$ (*reject*)

$\therefore$ *No solution*

⑤ NPV: $t \neq -1$, LCD: $t+1$

$t(t+1) - 5 = 3(t+1)$

$t^2 + t - 5 = 3t + 3$

$t^2 - 2t - 8 = 0$

$(t-4)(t+2) = 0$

$t = 4,\ -2$

⑥ NPV: $y \neq 1$, LCD: $y-1$

$y(y-1) + 3 = 3y$

$y^2 - y + 3 = 3y$

$y^2 - 4y + 3 = 0$

$(y-1)(y-3) = 0$

$y = \not{1}$ (*reject*), 3

$\therefore\ y = 3$

⑦ NPV: $x \neq -2$, LCD: $x+2$

$2x = (x+1)(x+2) - 4$

$2x = x^2 + 3x + 2 - 4$

$0 = x^2 + x - 2$

$0 = (x-1)(x+2)$

$x = 1,\ \not{-2}$ (*reject*)

$\therefore\ x = 1$

[EX3] Multiply both sides by LCD.

① NPV: $n \neq \pm 2$, LCD: $(n-2)(n+2)$

$2n(n+2) - (n-2) = 2(n-2)(n+2)$

$2n^2 + 4n - n + 2 = 2(n^2 - 4)$

$2n^2 + 4n - n + 2 = 2n^2 - 8$

$3n = -10$

$n = -\frac{10}{3}$

② NPV: $x \neq 2,\ 3$, LCD: $(x-3)(x-2)$

$4(x-2) - 6(x-3) = (x-3)(x-2)$

$4x - 8 - 6x + 18 = x^2 - 5x + 6$

$-2x + 10 = x^2 - 5x + 6$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4,\ -1$

③ NPV: $x \neq 0,\ -2$, LCD: $x(x+2)$

$x + 2 - 4 = 2x$

$x - 2 = 2x$

$-2 = x$

$x = \not{-2}$ (*reject*)

$\therefore$ *No solution*

④ NPV: $x \neq \pm 3$, LCD: $(x+3)(x-3)$

$x(x-3) + x(x+3) = 2(x+3)(x-3)$

$x^2 - 3x + x^2 + 3x = 2(x^2 - 9)$

$2x^2 = 2x^2 - 18$

$0 = -18$

$\therefore$ *No solution*

⑤ NPV: $x \neq \pm 1$, LCD: $(x+1)(x-1)$

$x - 5 + 2x(x+1) = (x+1)(x-1)$

$x - 5 + 2x^2 + 2x = x^2 - 1$

$2x^2 + 3x - 5 = x^2 - 1$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$x = -4,\ \not{1}$ (*reject*)

$\therefore\ x = -4$

⑥ NPV: $x \neq 1,\ 2$, LCD: $(x-1)(x-2)$

$2(x-2) - (x-1)^2 = 2x(x-2)$

$2x - 4 - (x^2 - 2x + 1) = 2x^2 - 4x$

$2x - 4 - x^2 + 2x - 1 = 2x^2 - 4x$

$-x^2 + 4x - 5 = 2x^2 - 4x$

$3x^2 - 8x + 5 = 0$

$(x-1)(3x-5) = 0$

$x = \not{1}$ (*reject*), $5/3$

$\therefore\ x = \frac{5}{3}$

⑦ NPV: $x \neq 2$, LCD: $x-2$

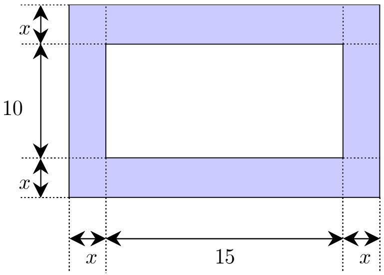
$x = (x-2) + 2$

$x = x - 2 + 2$

$x = x$

$\therefore$ *All real numbers except* $x = 2$

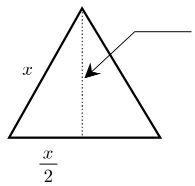
[EX16] Let x be the width of the walkaway.



Area of the walkaway $= \frac{1}{2} \times$ Area of the pool.

$$(10+2x)(15+2x) - 10\cdot 15 = \frac{1}{2}\cdot 10\cdot 15$$
$$(10+2x)(15+2x) = 150 + 75$$
$$150 + 50x + 4x^2 = 225$$
$$4x^2 + 50x - 75 = 0$$
$$x = \frac{-50 \pm \sqrt{50^2 - 4(4)(-75)}}{8} \approx 1.4,\ \ -13.9\ (reject) \qquad \therefore\ 1.4\ cm$$

[EX17] Let x be the side of the square.



Using the Pythagorean theorem,

$$h = \sqrt{x^2 - \left(\frac{x}{2}\right)^2} = \sqrt{x^2 - \frac{x^2}{4}} = \sqrt{\frac{3}{4}x^2} = \frac{\sqrt{3}}{2}x$$

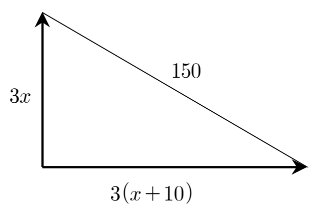
Area of triangle $= \frac{1}{2}\cdot x\cdot\frac{\sqrt{3}}{2}x = \frac{\sqrt{3}}{4}x^2$

Area of triangle + Area of rectangle $= 20$

$$\frac{\sqrt{3}}{4}x^2 + x^2 = 20$$
$$\sqrt{3}x^2 + 4x^2 = 80$$
$$(\sqrt{3}+4)x^2 = 80 \ \Rightarrow\ x^2 = \frac{80}{\sqrt{3}+4}$$
$$\therefore\ x = \sqrt{\frac{80}{\sqrt{3}+4}} \approx 3.736$$
$$\therefore\ Perimeter = 5x \approx 18.7\ m$$

[EX18] Let x be the speed of the slower train in km/h.

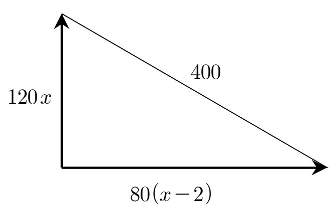
	Speed (mph)	Time (h)	Distance (miles)
Fast	$x+10$	3	$3(x+10)$
Slow	x	3	$3x$



$$(3x)^2 + (3(x+10))^2 = 150^2$$
$$9x^2 + 9(x+10)^2 = 22500$$
$$x^2 + (x+10)^2 = 2500$$
$$x^2 + x^2 + 20x + 100 = 2500$$
$$2x^2 + 20x - 2400 = 0$$
$$x^2 + 10x - 1200 = 0$$
$$(x+40)(x-30) = 0$$
$$x = -40\ (reject) \quad or \quad 30$$
$$\therefore\ Slow\ train:\ 30\ mph, \quad Fast\ train:\ 40\ mph$$

[EX19] Let x represent the time train A travels at 120 km/h.

	Speed (km/h)	Time (h)	Distance (km)
Train A	120	x	$120x$
Train B	80	$x-2$	$80(x-2)$



$$(120x)^2 + (80(x-2))^2 = 400^2$$
$$14400x^2 + 6400(x-2)^2 = 160000$$

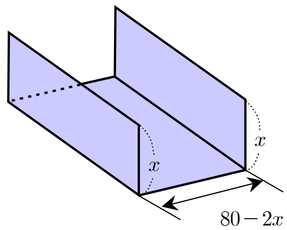
Divide both sides by 1600.

$$9x^2 + 4(x-2)^2 = 100$$
$$9x^2 + 4(x^2 - 4x + 4) = 100$$
$$13x^2 - 16x + 16 - 100 = 0$$
$$13x^2 - 16x - 84 = 0$$
$$(x+2)(13x-42) = 0$$
$$x = -2\ (reject) \quad or \quad \frac{42}{13}$$
$$\therefore\ x = \frac{42}{13} \approx 3.2307 \approx 3hr\ and\ 14\ min$$
$$\therefore\ 3:14PM$$

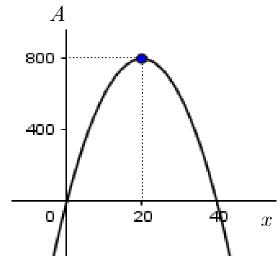
[EX20] Let x be the integer.

$$x + x^2 < 12 \quad and \quad x > 0$$
$$x^2 + x - 12 < 0$$
$$(x+4)(x-3) < 0$$
$$-4 < x < 3 \quad and \quad x > 0 \quad \Rightarrow \quad 0 < x < 3,\ x \in Z$$
$$\therefore\ x = 1,\ 2$$

[EX7] Let x be the height of the trough.

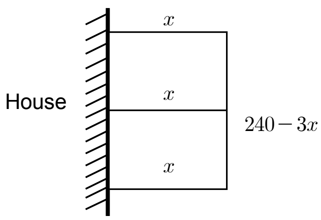


$$
\begin{aligned}
A &= x(80-2x)\\
&= 80x-2x^2\\
&= -2(x^2-40x+400-400)\\
&= -2(x^2-40x+400)+800\\
&= -2(x-20)^2+800
\end{aligned}
$$

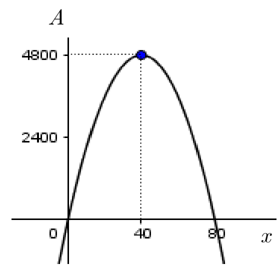


$\therefore\ 20\ cm$

[EX8] Let x be the side length of the garden.



$$
\begin{aligned}
A &= x(240-3x)\\
&= 240x-3x^2\\
&= -3(x^2-80x+1600-1600)\\
&= -3(x^2-80x+1600)+4800\\
&= -3(x-40)^2+4800
\end{aligned}
$$



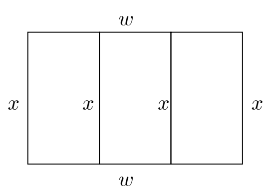
Maximum Area $= 4800\ m^2$

Dimensions: If $x=40,\ 240-3(40)=120.$

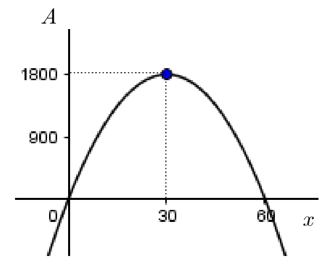
$\therefore\ 40\ m$ by $120\ m$

[EX9] Let x and w be the length and width of the rectangular pasture.

$$
\begin{aligned}
4x+2w&=240\\
2x+w&=120\\
w&=120-2x
\end{aligned}
$$



$$
\begin{aligned}
A &= x(120-2x)\\
&= 120x-2x^2\\
&= -2(x^2-60x+900-900)\\
&= -2(x^2-60x+900)+1800\\
&= -2(x-30)^2+1800
\end{aligned}
$$



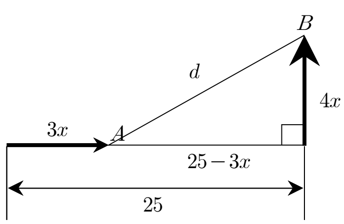
Maximum Area $= 1800\ m^2$

Dimensions: If $x=30,\ w=120-2(30)=60.$

$\therefore\ 30\ m$ by $60\ m$

[EX10] Let x represent the time the ships travel.

	Speed (mph)	Time (h)	Distance (miles)
Ship A	3	x	$3x$
Ship B	4	x	$4x$



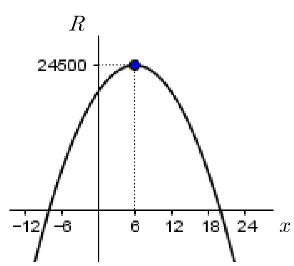
$$
\begin{aligned}
d^2 &= (25-3x)^2+(4x)^2\\
d &= \sqrt{(25-3x)^2+(4x)^2}\\
&= \sqrt{625-150x+9x^2+16x^2}\\
&= \sqrt{25x^2-150x+625}\\
&= \sqrt{25(x^2-6x+9-9)+625}\\
&= \sqrt{25(x^2-6x+9)-225+625}\\
&= \sqrt{25(x-3)^2+400}
\end{aligned}
$$

When $x=3$, d has a minimum distance of $\sqrt{0+400}=20.$

$\therefore$ The two ships are closest to each other at $3:00$ PM, and the distance is 20 miles.

[EX11] Let x be the number of \$0.5 decreases.

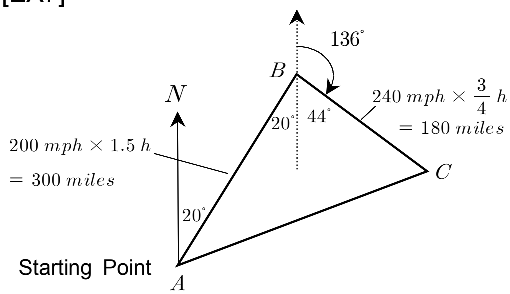
$$
\begin{aligned}
R &= (10-0.5x)(2000+250x)\\
&= 20000+2500x-1000x-125x^2\\
&= -125x^2+1500x+20000\\
&= -125(x^2-12x+36-36)+20000\\
&= -125(x^2-12x+36)+4500+20000\\
&= -125(x-6)^2+24500
\end{aligned}
$$



Maximum revenue $= \$24\,500$

Ticket price $=10-0.5x=10-0.5(6)=\$7$

[EX7]



In $\triangle ABC$, $AC = \sqrt{300^2 + 180^2 - 2(300)(180)\cos 64^\circ} \approx 273.963$

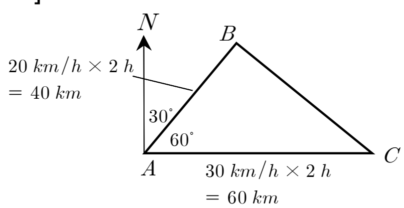
$$\frac{273.963}{\sin 64^\circ} = \frac{180}{\sin A}$$

$$\sin A = \frac{180 \sin 64^\circ}{273.963} \approx 0.5905$$

$\angle BAC = \sin^{-1}(0.5905) \approx 36.2^\circ$ $\quad\therefore$ $bearing = 20^\circ + 36.2^\circ = 56.2^\circ$

$\therefore$ The distance AC is approximately $274.0\,miles$.
The bearing of C from A is approximately 56.2°.

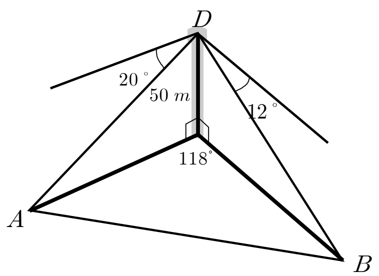
[EX8]



In $\triangle ABC$, $BC = \sqrt{40^2 + 60^2 - 2(40)(60)\cos 60^\circ} \approx 52.915$

$\therefore$ The distance between the two ships is approximately $52.9\,km$.

[EX9]



In $\triangle OAD$, $\angle ODA = 90^\circ - 20^\circ = 70^\circ$

$$\tan 70^\circ = \frac{OA}{50} \Rightarrow OA = 50\tan 70^\circ$$

In $\triangle OBD$, $\angle ODB = 90^\circ - 12^\circ = 78^\circ$

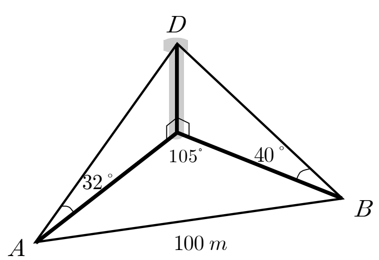
$$\tan 78^\circ = \frac{OB}{50} \Rightarrow OB = 50\tan 78^\circ$$

In $\triangle OAB$,

$AB^2 = (50\tan 70^\circ)^2 + (50\tan 78^\circ)^2 - 2(50\tan 70^\circ)(50\tan 78^\circ)(\cos 118^\circ) = 104\,547.072...$

$AB = \sqrt{104\,547.072...} \approx 323.3\,m$

[EX10]



Let $h = OD$.

In $\triangle OAD$, $\angle ODA = 90^\circ - 32^\circ = 58^\circ$

$$\tan 58^\circ = \frac{OA}{h} \Rightarrow OA = h\tan 58^\circ$$

In $\triangle OBD$, $\angle ODB = 90^\circ - 40^\circ = 50^\circ$

$$\tan 50^\circ = \frac{OB}{h} \Rightarrow OB = h\tan 50^\circ$$

In $\triangle OAB$,

$100^2 = (h\tan 58^\circ)^2 + (h\tan 50^\circ)^2 - 2(h\tan 58^\circ)(h\tan 50^\circ)(\cos 105^\circ)$

Factor out h^2.

$$10000 = h^2\left[\ (\tan 58^\circ)^2 + (\tan 50^\circ)^2 - 2(\tan 58^\circ)(\tan 50^\circ)(\cos 105^\circ)\ \right]$$

$$h = \frac{10000}{(\tan 58^\circ)^2 + (\tan 50^\circ)^2 - 2(\tan 58^\circ)(\tan 50^\circ)(\cos 105^\circ)} = 2012.6439...$$

$h = \sqrt{2012.6439...} \approx 44.9\,m$

[EX11]

$(2x+1)^2 = 8^2 + (x+2)^2 - 2\cdot 8\cdot(x+2)\cdot\cos 60^\circ$

$4x^2 + 4x + 1 = 64 + x^2 + 4x + 4 - 8(x+2)$

$4x^2 + 4x + 1 = x^2 - 4x + 52$

$3x^2 + 8x - 51 = 0$

$(3x+17)(x-3) = 0$

$x = -\frac{17}{3}\,(reject),\ \ 3$ $\quad\therefore$ $x = 3$

★ Ambiguous Case (SSA): p307

[EX1]

① $h = 12\sin 50^\circ = 9.2 > 7$
$\therefore$ No triangle

② $h = 10\sin 35^\circ = 5.7 < 8 < 10$
$\therefore$ 2 triangles

③ $h = 12\sqrt{3}\sin 60^\circ = 18 = a$
$\therefore$ 1 triangle

④ $a > b$
$\therefore$ 1 triangle

⑤ $b > c$
$\therefore$ 1 triangle

⑥ $\angle B$ is obtuse and $b < a$
$\therefore$ No triangle

⑦ $h = 20\sin 42^\circ = 13.4 < 15 < 20$
$\therefore$ 2 triangles

⑧ $\angle C$ is obtuse and $c < b$
$\therefore$ No triangle

At all levels of mathematics, the best way to learn new concepts is by solving numerous questions. Through this Pre-calculus 11 workbook, students master the mathematical foundations required to succeed in Calculus.

Following a short review of the fundamental concepts and guided examples, students are challenged with exercise questions ranging in difficulty. There are over 450 guided examples and 3000 questions, with full solutions, because practice makes perfect.

Chapter 1. Polynomials

Chapter 2. Radical Expressions and Equations

Chapter 3. Rational Expressions, Equations, and Inequalities

Chapter 4. Quadratic Equations and Inequalities

Chapter 5. Quadratic Functions

Chapter 6. Systems of Equations and Inequalities

Chapter 7. Trigonometry

Chapter 8. Absolute Value, Reciprocal, and Exponential Functions

Chapter 9. Financial Literacy

www.infinitechallenge.ca

ISBN 978-0-9916987-4-5

9 780991 698745 >

CAD $37.50