

Infinite Challenge

PRE-CALCULUS 10

Richard N.S. Seong
Philip Seong

- ✓ Key points concisely summarized
- ✓ More than 400 complete guided examples
- ✓ More than 3300 carefully crafted questions with full solutions!
- ✓ 1200 graphs and diagrams to help explain and solve problems!

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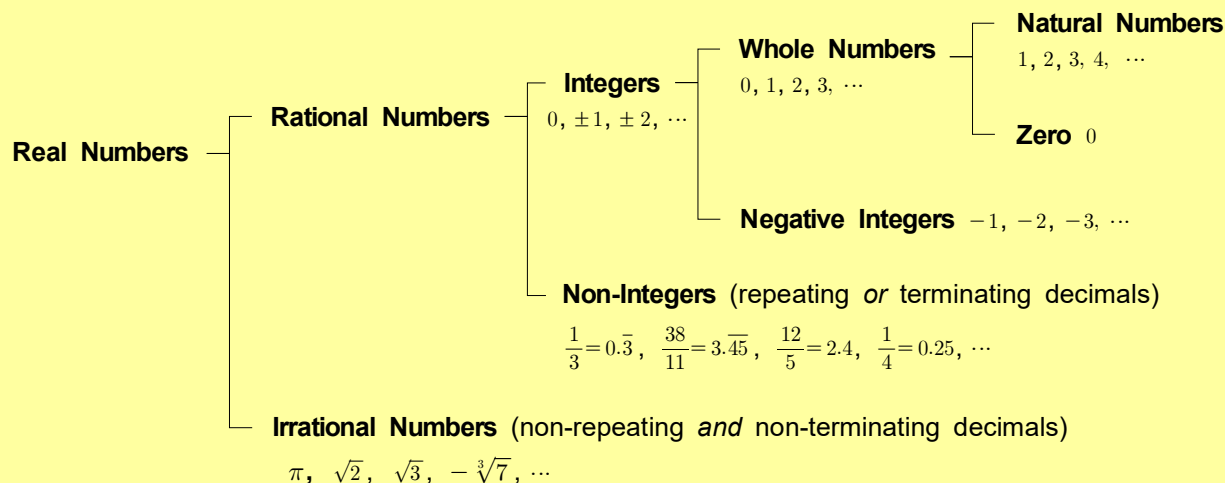
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1. Real Numbers

★ Real Number System

◆ Real Number System



- **Rational Numbers (Q):** Any real numbers that can be written in the form $\frac{p}{q}$. ($p \in \mathbb{Z}$, $q \in \mathbb{Z}$ and $q \neq 0$)

(e.g.) $-0.7 = -\frac{7}{10}, \quad 0.\bar{3} = \frac{1}{3}, \quad 6 = \frac{6}{1}, \quad -5 = \frac{-10}{2}$

- **Irrational Numbers (Q'):** Any real numbers that cannot be written in the form $\frac{p}{q}$.

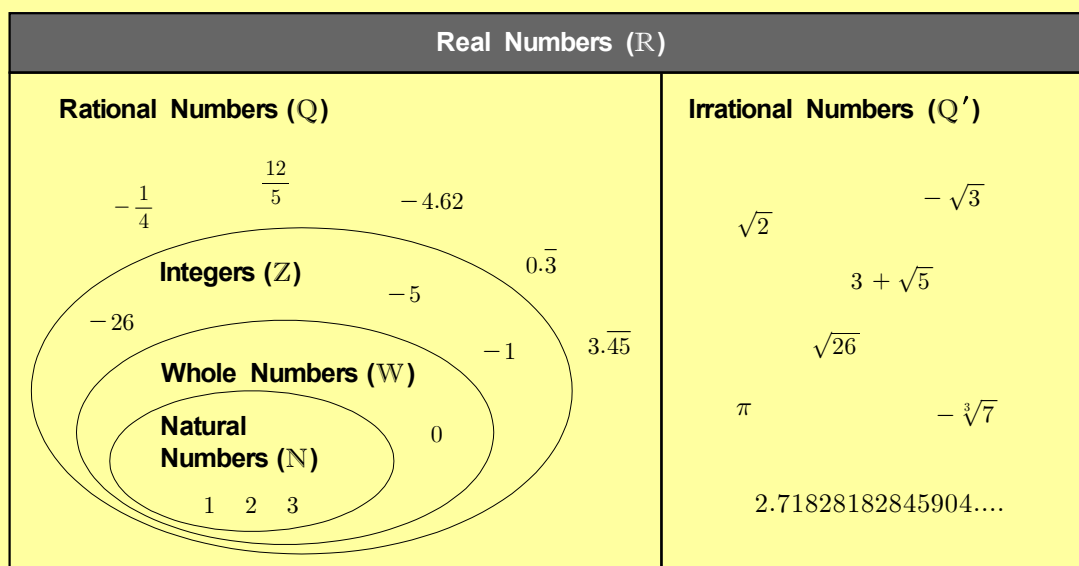
Irrational numbers are non-repeating and non-terminating decimals.

(e.g.) $\pi = 3.141592653\dots, \quad \sqrt{2} = 1.414213562\dots, \quad \sqrt{3} = 1.732050807\dots$

- **Integers (Z):** $\{0, \pm 1, \pm 2, \pm 3, \pm 4, \dots\}$

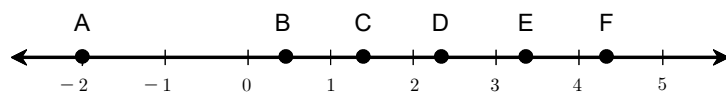
- **Whole Numbers (W):** $\{0, 1, 2, 3, 4, \dots\}$

- **Natural Numbers (N):** $\{1, 2, 3, 4, \dots\}$

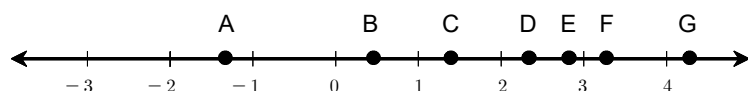


[EX12] Match each number with its corresponding point on the number line. Then arrange the numbers in order from least to greatest. Do not use a calculator.

① $\sqrt{6}$, $\sqrt[3]{-8}$, $\sqrt{20}$, $\sqrt[3]{40}$, $\sqrt[4]{5}$



② $\sqrt{11}$, $\sqrt[3]{23}$, $\sqrt{2}$, $\sqrt[3]{-3}$, $\sqrt[4]{30}$



[EX13] Determine whether each number is rational or irrational.

① $\sqrt{9}$

② $\sqrt[3]{-27}$

③ $\sqrt{6400}$

④ $\sqrt{125}$

⑤ $\sqrt[3]{\frac{64}{125}}$

⑥ $\sqrt[3]{0.27}$

⑦ $\sqrt{\frac{25}{169}}$

⑧ $\sqrt{432}$

[EX14] Given $\sqrt{2} \approx 1.41$ and $\sqrt{5} \approx 2.23$, estimate the value of the following without using a calculator.

① $\sqrt{200}$

② $\sqrt{0.02}$

③ $\sqrt{20}$

◆ Removing Brackets

$$+(A - B - C) = A - B - C$$

$$-(A - B - C) = -A + B + C$$

(Example 4) Simplify.

① $(7x^2 - 8x + 12) + (3x^2 - 4x + 5)$

(Method 1) Horizontal Addition

$$\begin{aligned} &= (7x^2) - 8x + 12 + (3x^2) - 4x + 5 \\ &= 10x^2 - 12x + 17 \end{aligned}$$

(Method 2) Vertical Addition

$$\begin{array}{r} (7x^2) - 8x + 12 \\ + (3x^2) - 4x + 5 \\ \hline = 10x^2 - 12x + 17 \end{array}$$

② $(5a^2 + 4b - 13) - (2a^2 - 3b - 6)$

(Method 1) Horizontal Subtraction

$$\begin{aligned} &= (5a^2) + 4b - 13 - (2a^2) + 3b + 6 \\ &= 3a^2 + 7b - 7 \end{aligned}$$

(Method 2) Vertical Subtraction

$$\begin{array}{r} (5a^2) + 4b - 13 \\ - (2a^2) - 3b - 6 \\ \hline = 3a^2 + 7b - 7 \end{array}$$

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[EX8] Simplify.

① $(6x - 2y) + (-7x + 11y)$

② $(16ab - 5a) - (8a + 13ab)$

③ $(2m - 4n) - (5n - m)$

④ $-(11x + 15) + (15x + 9)$

⑤ $(3x + 4y - 5) + (-5x + 7y - 8)$

⑥ $(5x^2 - 8x + 2) - (-12x^2 + 6x - 3)$

⑦ $(-4a^2 + 3a + 2) - (a^2 - 2a - 3)$

⑧ $(2x^2 - 3x + 7) + (-3x^2 + 6x - 4)$

⑲ $(xy + 6)(xy - 3)$

⑳ $(x^2 + 4)(4x^2 + 9)$

㉑ $(t^3 - 8)(t^3 + 2)$

㉒ $(8p^3 - 1)(p^3 + 8)$

㉓ $(3a^2 - 5b^2)(5a^2 - 2b^2)$

㉔ $(4x^2 + 5y)(7x^2 + 6y)$

㉕ $(2m^2 - 3n^2)(4m^2 - 3n^2)$

㉖ $\left(2x + \frac{y}{3}\right)\left(3x - \frac{y}{2}\right)$

㉗ $\left(4a - \frac{b}{5}\right)\left(5a + \frac{b}{4}\right)$

(Example 6) Expand and simplify:

① $3x(3x - 4)(x + 5)$

$= 3x(3x^2 + 11x - 20)$

$= 9x^3 + 33x^2 - 60x$

Multiply the two binomials first.
Then multiply the result by $3x$.

② $(2x + 3)^3$

$$= (2x + 3)(2x + 3)^2 = (2x + 3)(4x^2 + 12x + 9) = 8x^3 + 24x^2 + 18x + 12x^2 + 36x + 27$$

$$= 8x^3 + 36x^2 + 54x + 27$$

[EX8] Expand and simplify.

① $2x(x - 3)(2x + 5)$

② $3(2x + 7)(5x - 4)$

③ $(x - y)(x + y)^2$

(Example 2) Factor.

① $2x^2 + 5x - 3$

$$\begin{array}{r} \boxed{2} \rightarrow 3 \\ \boxed{1} \rightarrow -1 \end{array} \rightarrow \frac{-2}{1} \times$$

$$\begin{array}{r} \boxed{2} \rightarrow -3 \\ \boxed{1} \rightarrow 1 \end{array} \rightarrow \frac{2}{-1} \times$$

$$\begin{array}{r} \boxed{2} \rightarrow 1 \\ \boxed{1} \rightarrow -3 \end{array} \rightarrow \frac{-6}{-5} \times$$

$$\begin{array}{r} \boxed{2} \rightarrow -1 \\ \boxed{1} \rightarrow 3 \end{array} \rightarrow \frac{6}{5} \checkmark$$

$$\therefore 2x^2 + 5x - 3 = (2x-1)(x+3)$$

③ $2x^2 - x - 3$

$$\begin{array}{r} \boxed{2} \rightarrow -3 \\ \boxed{1} \rightarrow 1 \end{array} \rightarrow \frac{2}{-1} \checkmark$$

$$= (2x-3)(x+1)$$

④ $6x^2 - 5xy - 6y^2$

$$\begin{array}{r} \boxed{2} \rightarrow -3 \\ \boxed{3} \rightarrow 2 \end{array} \rightarrow \frac{4}{-5} \checkmark$$

$$= (2x-3y)(3x+2y)$$

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[EX2] Factor.

① $2x^2 - 3x - 2$

② $2x^2 + 3x - 5$

③ $2a^2 - a - 6$

④ $3x^2 + 5x - 2$

⑤ $3x^2 - x - 2$

⑥ $3m^2 - 2m - 5$

⑦ $-4x^2 - x + 3$

⑧ $4y^2 + 4y - 3$

⑨ $-5t^2 + 3t + 8$

3. Relations and Functions

★ Relations

◆ Relation

A relation is a set of ordered pairs (*input, output*) or (x, y) .

◆ Domain

The set of the input values or x values in the ordered pairs is called the **domain** of the relation. x is called the **independent variable**.

◆ Range

The set of the output values or y values in the ordered pairs is called the **range** of the relation. y is called the **dependent variable** because its value depends on the value of x .

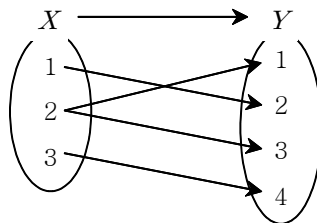
(Example 1) Consider the relation $\{(1, 2), (2, 1), (3, 4), (2, 3)\}$.

① Express the relation:

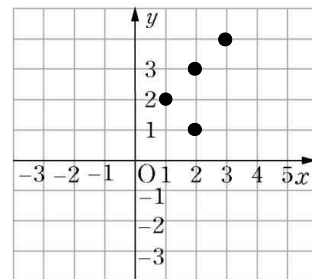
(1) as a table

| x | y |
|-----|-----|
| 1 | 2 |
| 2 | 1 |
| 3 | 4 |
| 2 | 3 |

(2) as an arrow diagram



(3) as a graph



② Find the domain and range.

$$\text{Domain} = \{1, 2, 3\}, \quad \text{Range} = \{1, 2, 3, 4\}$$

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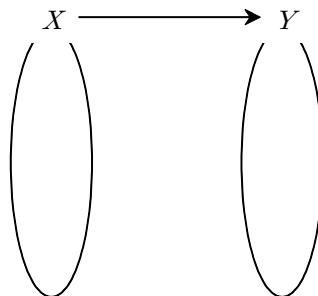
[EX1] Consider the relation $\{(-2, -2), (0, 1), (3, 1), (5, 3)\}$.

① Express the relation:

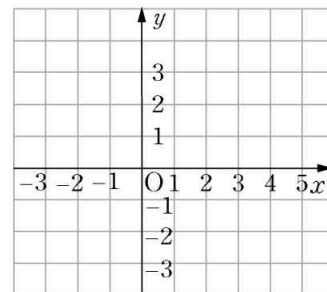
(1) as a table

| x | y |
|-----|-----|
| | |
| | |
| | |
| | |

(2) as an arrow diagram

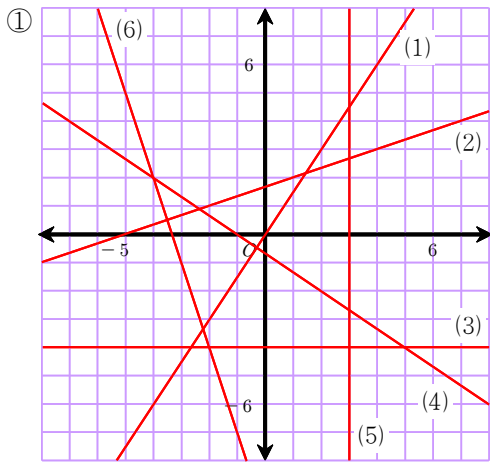


(3) as a graph



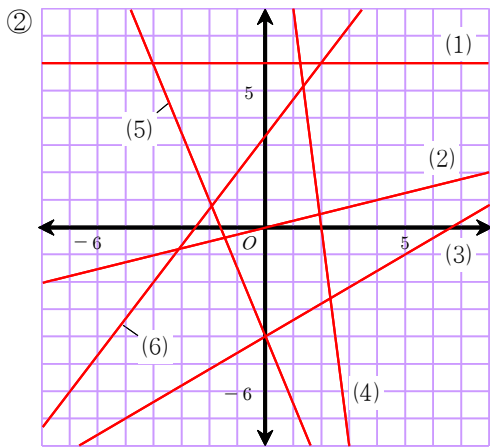
② Find the domain and range.

[EX4] Find the slope of each line.



(1) _____ (2) _____ (3) _____

(4) _____ (5) _____ (6) _____

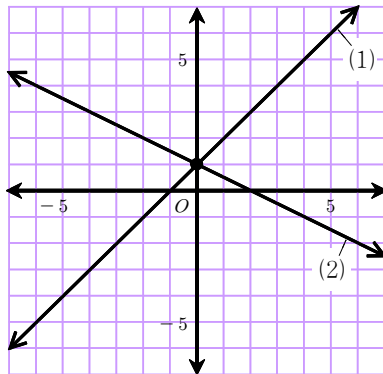


(1) _____ (2) _____ (3) _____

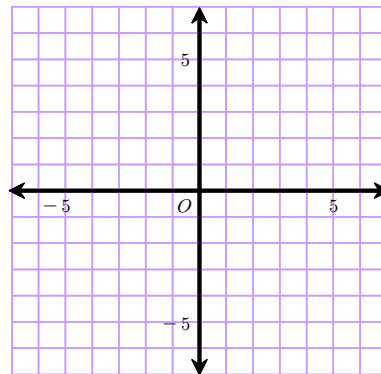
(4) _____ (5) _____ (6) _____

[EX5] Graph the line passing through each point with each given slope.

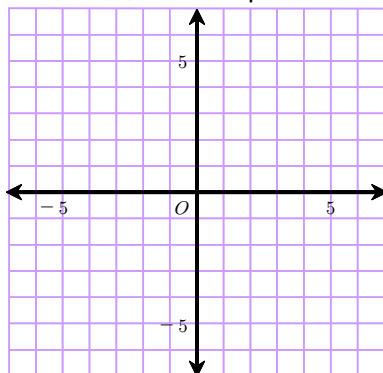
① (0, 1) with slope (1) 1 (2) $-\frac{1}{2}$



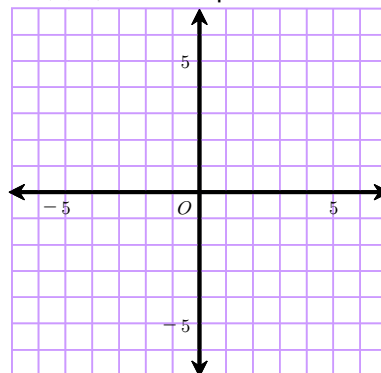
② (0, 0) with slope (1) 2 (2) -1



③ (0, -1) with slope (1) 1 (2) 3



④ (2, 0) with slope (1) 2 (2) -2



★ Point-Slope Form of Linear Equations, $y - y_1 = m(x - x_1)$

The equation of a line through (x_1, y_1) with a slope of m is $y - y_1 = m(x - x_1)$.

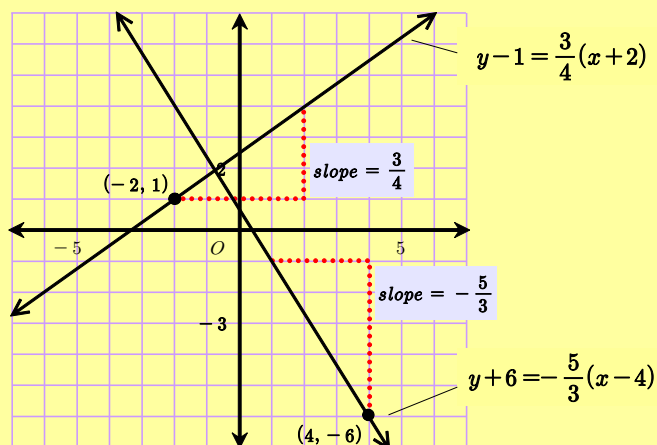
$$y - y_1 = m(x - x_1)$$

Point

Slope

◆ Slope

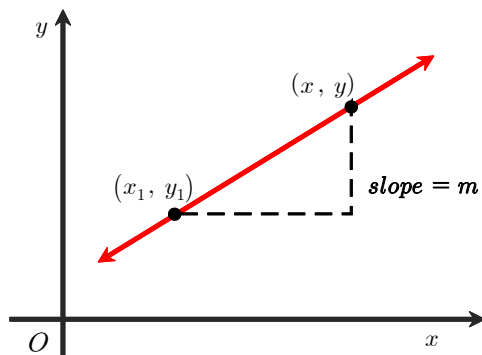
- If $m > 0$, the line rises from left to right.
- If $m < 0$, the line falls from left to right.



[Proof]

Consider a line through (x_1, y_1) with a slope of m .

Let (x, y) be an arbitrary point on the line.



$$\text{Slope of the line: } m = \frac{y - y_1}{x - x_1}$$

$$\text{Multiply both sides by } x - x_1: m(x - x_1) = y - y_1$$

$$\therefore y - y_1 = m(x - x_1)$$

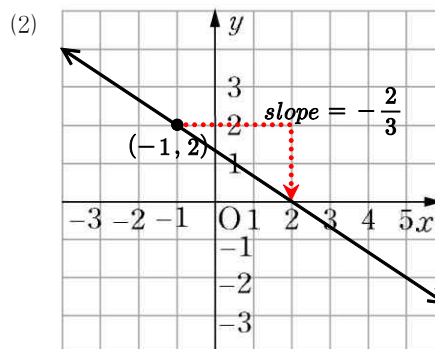
(Example 1) Consider the equation $y - 2 = -\frac{2}{3}(x + 1)$.

(1) Find the slope of the line and identify a point on the line. (2) Graph the line.

$$(1) y - 2 = -\frac{2}{3}(x + 1) \quad \Leftarrow \quad y - y_1 = m(x - x_1)$$

$$m = -\frac{2}{3}, \quad (x_1, y_1) = (-1, 2)$$

$$\therefore \text{Slope} = -\frac{2}{3}, \quad \text{Point: } (-1, 2)$$



(Example 2) Maya earns a monthly base salary plus a commission on each beauty product she sells. In one month, she sold \$14 500 worth of products and earned \$3034. The following month, her sales totaled \$21 750, and her earnings were \$4049.

① Find her commission rate.

⇒ The line passes through the points (14 500, 3034) and (21 750, 4049).

$$\frac{4049 - 3034}{21\,750 - 14\,500} = \frac{1015}{7250} = 0.14 \quad \therefore \text{Maya's commission rate is } 14\%.$$

② Write the equation representing Maya's monthly earnings E , in dollars, in terms of her product sales x , in dollars.

⇒ Write the equation in point-slope form: $E - 3034 = 0.14(x - 14\,500)$

$$E - 3034 = 0.14x - 2030$$

$$E = 0.14x - 2030 + 3034$$

$$\therefore E = 0.14x + 1004$$

③ Find her monthly base salary.

⇒ When $x = 0$, $E = 0.14(0) + 1004 = 1004$

∴ Maya's monthly base salary is \$1004.

④ If her total sales this month is \$35 000, how much will she earn?

⇒ When $x = 35\,000$, $E = 0.14(35\,000) + 1004 = 7034$

∴ Maya will earn \$7034.

[EX14] A sneaker store employee earns a monthly base salary plus a commission on every pair of sneakers he sells. In March, he sold \$28 800 worth of sneakers, earning him \$4 448. The following month, he sold \$36 000 worth of sneakers and earned \$5 168.

① Find his commission rate.

② Write the equation representing the employee's earnings E , in dollars, in terms of his sales x , in dollars.

③ Find his monthly base salary.

④ If his total sales this month is \$30 000, how much will he earn?

★ Solving Systems of Linear Equations by Elimination

◆ Solving Systems of Equations by Elimination

Step 1. Rewrite both equations in standard form, $Ax + By = C$.

Step 2. Multiply one or more of the equations by a non-zero number to make the coefficients of one variable equal.

Step 3. Add or subtract the equations to eliminate one variable, then solve for the other variable.

Step 4. Substitute the solution into either original equation, then solve for the eliminated variable.

Step 5. Write the solution as an ordered pair.

(Example 1) Solve by elimination:
$$\begin{cases} 2x + y = 9 \\ x - 2y = 2 \end{cases}$$

(Method 1) **Addition**

$$\begin{array}{l} 2x + y = 9 \quad \xrightarrow{\text{(Multiply by 2)}} \\ x - 2y = 2 \quad \xrightarrow{\text{(Leave alone)}} \end{array} + \begin{array}{l} 4x + \cancel{2y} = 18 \\ x - \cancel{2y} = 2 \\ \hline 5x = 20 \\ x = 4 \end{array}$$

Add the equations to eliminate the variable y , then solve for x .

Substitute 4 for x in one of the original equations: $2(4) + y = 9$
 $y = 1$

\therefore The solution is $(4, 1)$.

(Method 2) **Subtraction**

$$\begin{array}{l} 2x + y = 9 \quad \xrightarrow{\text{(Leave alone)}} \\ x - 2y = 2 \quad \xrightarrow{\text{(Multiply by 2)}} \end{array} - \begin{array}{l} \cancel{2x} + y = 9 \\ \cancel{2x} - 4y = 4 \\ \hline 5y = 5 \\ y = 1 \end{array}$$

Subtract the equations to eliminate the variable x , then solve for y .

Substitute 1 for y in one of the original equations: $x - 2(1) = 2$
 $x = 4$

\therefore The solution is $(4, 1)$.

[EX1] Solve each system of equations by elimination.

①
$$\begin{cases} x + 2y = 11 \\ x - y = -4 \end{cases}$$

②
$$\begin{cases} 4x - 3y = 11 \\ 2x + 3y = 1 \end{cases}$$

(Example 5) A motorboat travels 48 km upstream in 3 hours and returns downstream in 2 hours. Find the speed of the motorboat in still water and the speed of the current.

Let x be the speed of the motorboat, and y be the speed of the current.

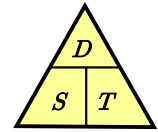
| | Distance (km) | Speed (kph) | Time (h) |
|------------|------------------|----------------|-------------|
| Upstream | 48 | $x - y$ | 3 |
| Downstream | 48 | $x + y$ | 2 |

$$D = S \cdot T$$

$$3(x - y) = 48 \Rightarrow x - y = 16 \quad \textcircled{1}$$

$$2(x + y) = 48 \Rightarrow x + y = 24 \quad \textcircled{2}$$

$$\begin{array}{r} \textcircled{1} + \textcircled{2} \\ + \\ \hline \end{array} \left| \begin{array}{l} x - y = 16 \\ x + y = 24 \\ \hline 2x = 40 \end{array} \right. \begin{array}{l} \therefore x = 20 \\ \textcircled{1} \quad 20 - y = 16 \end{array} \therefore y = 4$$



\therefore The speed of the boat in still water is 20 km/h and the speed of the current is 4 km/h.

[EX5] Solve each of the following problems. Show your work.

① A plane travels 3360 km from Vancouver to Toronto in 5 hours with a tail wind. The return trip takes 6 hours with a head wind. If the wind speed is constant, find the speed of the plane in still air and the speed of the wind.

② Two motorboats are 280 km apart and are traveling toward each other. One boat's speed is 40 km/h faster than the other. If they meet after 2 hours, find the speed of each motorboat.

③ Philip traveled 970 km from Vancouver to Calgary, by car and train. He traveled part of the journey by car at 80 km/h, then took a train for the remaining distance at 100 km/h. If the entire trip took 10.5 hours, how many hours did he spend driving, and how many did he spend on the train?

★ Arithmetic Series

◆ A **series** is the sum of all terms in a sequence.

(e.g.) 1, 3, 5, 7, 9, ... is a sequence.

1 + 3 + 5 + 7 + 9 + ... is a series.

The sum of the first n terms of a series is denoted by S_n .

$$S_1 = t_1$$

$$S_2 = t_1 + t_2$$

$$S_3 = t_1 + t_2 + t_3$$

$$S_4 = t_1 + t_2 + t_3 + t_4$$

⋮

$$S_n = t_1 + t_2 + t_3 + t_4 + \dots + t_n$$

◆ An **arithmetic series** is the sum of the terms of an arithmetic sequence. The sum of the first n terms of an arithmetic series is given by

$$S_n = \frac{n(a + t_n)}{2} = \frac{n[2a + (n-1)d]}{2}$$

where S_n is the sum of the first n terms, t_n is the n th term, a is the first term, n is the number of terms, and d is the common difference.

[Proof]

Let S_{100} be the sum of the first 100 natural number.

$$S_{100} = 1 + 2 + 3 + 4 + \dots + 97 + 98 + 99 + 100$$

There are 50 pairs of 101.

$$\therefore S_{100} = \boxed{50} \times \boxed{101} = 5050$$

$$\frac{\text{Number of terms}}{2} \quad \uparrow \quad \uparrow \quad \text{(First term + Last term)}$$

The same method can be used to find S_n .

Let S_n be the sum of the first n terms of the arithmetic series.

$$S_n = a + (a+d) + (a+2d) + \dots + (t_n - 2d) + (t_n - d) + t_n \quad \text{where } t_n \text{ is the last term.}$$

$$\text{There are } \frac{n}{2} \text{ pairs of } (a+t_n). \quad \therefore S_n = \frac{n(a+t_n)}{2} \quad (1)$$

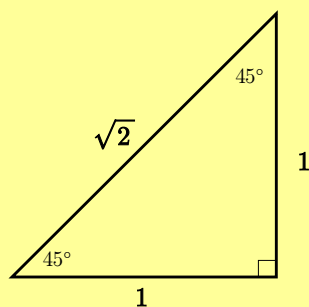
Substitute $t_n = a + (n-1)d$ for t_n in (1).

$$S_n = \frac{n(a+t_n)}{2} = \frac{n[a + a + (n-1)d]}{2} = \frac{n[2a + (n-1)d]}{2} \quad \therefore S_n = \frac{n(2a + (n-1)d)}{2} \quad (2)$$

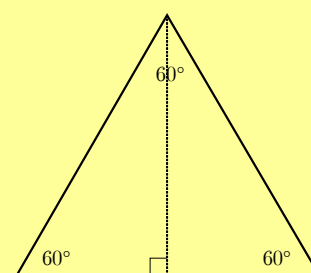
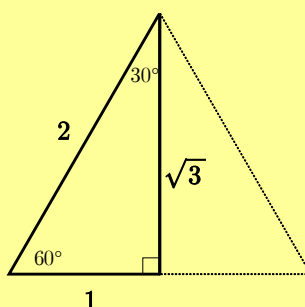
8. Right Triangle Trigonometry

★ Special Right Triangles

◆ 45°-45°-90° (Isosceles Right Triangle)

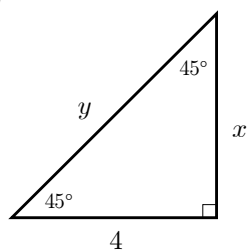


◆ 30°-60°-90° (Half an Equilateral Triangle)



(Example 1) Find the values of x and y , using the ratio of the sides.

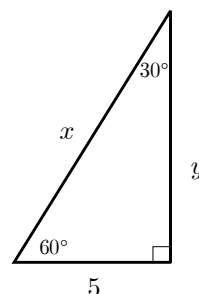
①



$$\frac{x}{4} = \frac{1}{1} \quad \therefore x = 4$$

$$\frac{y}{4} = \frac{\sqrt{2}}{1} \quad \therefore y = 4\sqrt{2}$$

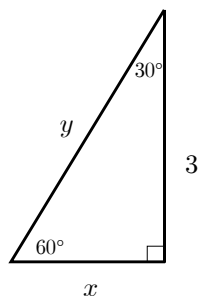
②



$$\frac{x}{5} = \frac{2}{1} \quad \therefore x = 10$$

$$\frac{y}{5} = \frac{\sqrt{3}}{1} \quad \therefore y = 5\sqrt{3}$$

③



$$\frac{x}{3} = \frac{1}{\sqrt{3}}$$

$$x = \frac{3}{\sqrt{3}} = \frac{3 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$$

(Rationalize denominator)

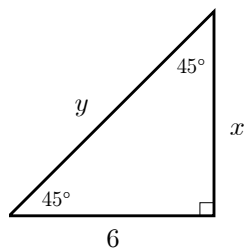
$$\therefore x = \sqrt{3}$$

$$\frac{y}{x} = \frac{2}{1} \Rightarrow y = 2x$$

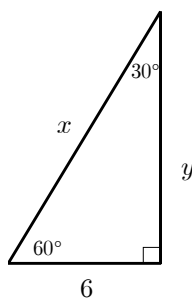
$$\therefore y = 2\sqrt{3}$$

[EX1] Find the values of x and y , using the ratio of the sides. Do not use a calculator.

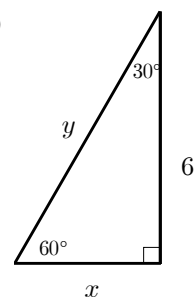
①



②



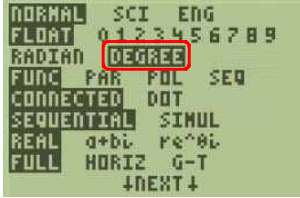
③



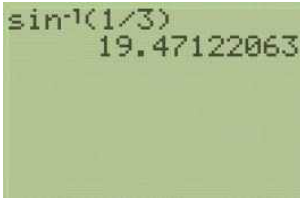
(Example 4) Use a calculator to solve for θ , to one decimal place. ($0^\circ \leq \theta \leq 90^\circ$).

$$\sin\theta = \frac{1}{3}$$

Step 1. Press **MODE**. Scroll down to **Radian Degree**, select **Degree**, then press **ENTER**.



Step 2. Press **2nd** **MODE**. Press **2nd** **SIN** and type $1/3$, then press **ENTER**.



$$\therefore \theta = 19.5^\circ$$

[EX5] Use a calculator to solve for θ , in degrees to one decimal place. ($0^\circ \leq \theta \leq 90^\circ$).

① $\sin\theta = \frac{3}{4}$

② $\cos\theta = 0.4728$

③ $\tan\theta = \frac{4}{7}$

$$\theta = \sin^{-1}\left(\quad\right)$$

$$\theta = \cos^{-1}\left(\quad\right)$$

$$\theta = \tan^{-1}\left(\quad\right)$$

$$\therefore \theta =$$

$$\therefore \theta =$$

$$\therefore \theta =$$

④ $\tan\theta = 1.4035$

⑤ $\cos\theta = \frac{2}{3}$

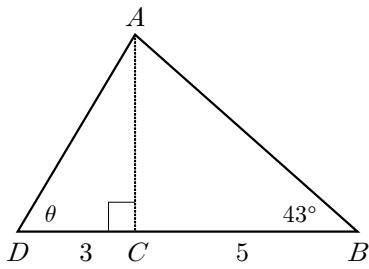
⑥ $\sin\theta = 0.5$

⑦ $\sin\theta = 0.4537$

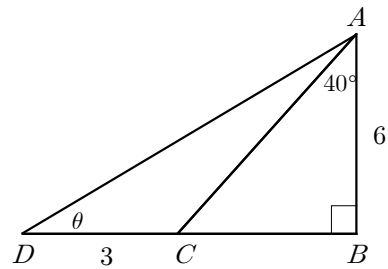
⑧ $\tan\theta = 4.2644$

⑨ $\cos\theta = 1$

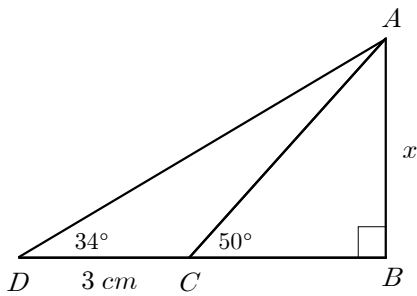
⑤



⑥



(Example 9) Use a calculator to find the length of side x in the diagram, to one decimal place.



In $\triangle ABC$, $\angle CAB = 40^\circ$.

In $\triangle ABD$, $\angle DAB = 56^\circ$.

$$\text{In } \triangle ABC, \tan 40^\circ = \frac{CB}{x} \Rightarrow CB = x \tan 40^\circ$$

$$\text{In } \triangle ABD, \tan 56^\circ = \frac{DB}{x} \Rightarrow DB = x \tan 56^\circ$$

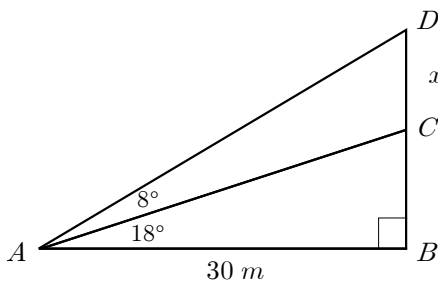
$$DB - CB = 3 \Rightarrow x \tan 56^\circ - x \tan 40^\circ = 3$$

$$\text{Factor out the } x. \quad x(\tan 56^\circ - \tan 40^\circ) = 3$$

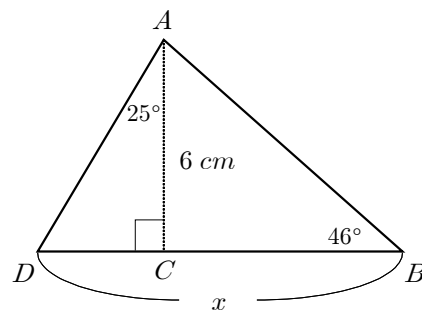
$$\therefore x = \frac{3}{\tan 56^\circ - \tan 40^\circ} = 4.66\dots \approx \underline{4.7 \text{ cm}}$$

[EX10] Use a calculator to find the length of side x in each diagram, to one decimal place.

①



②



★ Applications of Trigonometry

◆ Solving Applications of Trigonometry

Step 1. Read the problem carefully to identify the given informations (angles and sides) and the value to be found.

Step 2. Draw a clear diagram with the given informations and the unknown value.
If necessary, draw auxiliary lines to form a right triangle.

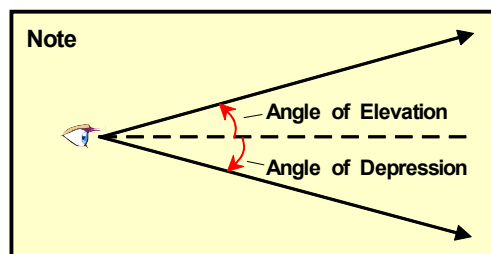
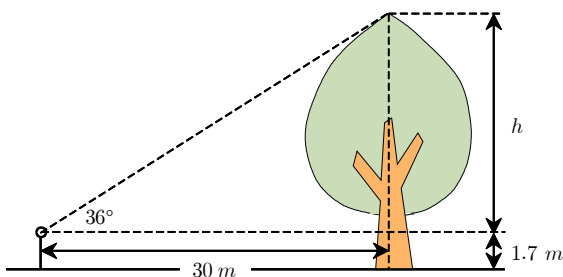
Step 3. Choose one of the trigonometric ratios: $\sin\theta$, $\cos\theta$, or $\tan\theta$.

Step 4. Setup and solve the equation.

Step 5. Check the solutions by using the original words of the problem.

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(Example 1) A student stands 30 m from the base of an oak tree. His eye level is 1.7 m above the ground and he measures the angle of elevation to the top of the tree to be 36° . Find the height of the oak tree, to the nearest tenth of a meter.



$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} \Rightarrow \tan 36^\circ = \frac{h}{30} \Rightarrow h = 30 \tan 36^\circ = 21.79\dots$$

$$\begin{aligned} \text{Height of the oak tree} &= \text{Eye level} + h \\ &= 1.7 + 21.79\dots \\ &= 23.49\dots \end{aligned}$$

\therefore The height of the oak tree is 23.5 m .

[EX1] – [EX24] Round your answer to the nearest tenth.

[EX1] A camera is placed on the ground 80 m from the base of a flagpole. If the angle of elevation to the top of the flagpole is 30° , find the height of the pole.

[EX2] From a fishing boat, the angle of elevation to the top of a cliff is 12° . If the distance from the boat to the base of the cliff is 320 m , find the height of the cliff.

SOLUTIONS

[EX9]

$$\begin{array}{l} \textcircled{1} \begin{array}{l} \textcircled{2} \overline{) 78 \quad 102} \\ \textcircled{3} \overline{) 39 \quad 51} \\ \hline 13 \quad 17 \end{array} \quad \therefore GCF = 2 \cdot 3 = 6 \\ \textcircled{2} \begin{array}{l} \textcircled{2} \overline{) 66 \quad 154} \\ \textcircled{11} \overline{) 33 \quad 77} \\ \hline 3 \quad 7 \end{array} \quad \therefore GCF = 2 \cdot 11 = 22 \\ \textcircled{3} \begin{array}{l} \textcircled{2} \overline{) 198 \quad 234} \\ \textcircled{9} \overline{) 99 \quad 117} \\ \hline 11 \quad 13 \end{array} \quad \therefore GCF = 2 \cdot 9 = 18 \\ \textcircled{4} \begin{array}{l} \textcircled{2} \overline{) 56 \quad 98 \quad 70} \\ \textcircled{7} \overline{) 28 \quad 49 \quad 35} \\ \hline 4 \quad 7 \quad 5 \end{array} \quad \therefore GCF = 2 \cdot 7 = 14 \\ \textcircled{5} \begin{array}{l} \textcircled{4} \overline{) 396 \quad 540 \quad 504} \\ \textcircled{9} \overline{) 99 \quad 135 \quad 126} \\ \hline 11 \quad 15 \quad 14 \end{array} \quad \therefore GCF = 4 \cdot 9 = 36 \end{array}$$

[EX10]

$$\begin{array}{llll} \textcircled{1} \quad 18 = 2 \cdot 3^2 & \textcircled{2} \quad 60 = 2^2 \cdot 3 \cdot 5 & \textcircled{3} \quad 75 = 3 \cdot 5^2 & \textcircled{4} \quad 12 = 2^2 \cdot 3 \\ \quad \quad 24 = 2^3 \cdot 3 & \quad \quad 48 = 2^4 \cdot 3 & \quad \quad 45 = 3^2 \cdot 5 & \quad \quad 15 = 1 \cdot 3 \cdot 5 \\ \therefore LCM = 2^3 \cdot 3^2 = 72 & \therefore LCM = 2^4 \cdot 3 \cdot 5 = 240 & \therefore LCM = 3^2 \cdot 5^2 = 225 & 18 = 2 \cdot 3^2 \quad \therefore LCM = 2^2 \cdot 3^2 \cdot 5 = 180 \end{array}$$

[EX11]

$$\begin{array}{lll} \textcircled{1} \begin{array}{l} \textcircled{6} \overline{) 54 \quad 60} \\ \textcircled{9} \overline{) 9 \quad 10} \\ \hline \textcircled{10} \end{array} \quad \therefore LCM = 6 \cdot 9 \cdot 10 = 540 \\ \textcircled{2} \begin{array}{l} \textcircled{8} \overline{) 64 \quad 80} \\ \textcircled{2} \overline{) 8 \quad 10} \\ \hline \textcircled{4} \quad \textcircled{5} \end{array} \quad \therefore LCM = 8 \cdot 2 \cdot 4 \cdot 5 = 320 \\ \textcircled{3} \begin{array}{l} \textcircled{8} \overline{) 72 \quad 96} \\ \textcircled{3} \overline{) 9 \quad 12} \\ \hline \textcircled{3} \quad \textcircled{4} \end{array} \quad \therefore LCM = 8 \cdot 3 \cdot 3 \cdot 4 = 288 \\ \textcircled{4} \begin{array}{l} \textcircled{3} \overline{) 24 \quad 45 \quad 36} \\ \textcircled{4} \overline{) 8 \quad 15 \quad 12} \\ \textcircled{3} \overline{) 2 \quad 15 \quad 3} \\ \hline \textcircled{2} \quad \textcircled{5} \quad \textcircled{1} \end{array} \quad \therefore LCM = 3 \cdot 4 \cdot 3 \cdot 2 \cdot 5 = 360 \\ \textcircled{5} \begin{array}{l} \textcircled{6} \overline{) 48 \quad 72 \quad 90} \\ \textcircled{4} \overline{) 8 \quad 12 \quad 15} \\ \textcircled{3} \overline{) 2 \quad 3 \quad 15} \\ \hline \textcircled{2} \quad \textcircled{1} \quad \textcircled{5} \end{array} \quad \therefore LCM = 6 \cdot 4 \cdot 3 \cdot 2 \cdot 5 = 720 \end{array}$$

[EX12]

$$\textcircled{1} = \frac{3}{4} \quad \textcircled{2} = \frac{5}{6} \quad \textcircled{3} = \frac{7}{3} \quad \textcircled{4} = \frac{11}{19} \quad \textcircled{5} = \frac{3}{4} \quad \textcircled{6} = \frac{17}{19}$$

[EX13]

$$\textcircled{6} \overline{) 48 \quad 36 \quad 30} \\ \quad \quad 8 \quad 6 \quad 5 \quad \therefore GCF = 6 \text{ baskets}$$

[EX14]

$$\textcircled{4} \overline{) 288 \quad 144 \quad 160} \\ \textcircled{4} \overline{) 72 \quad 36 \quad 40} \\ \quad \quad 18 \quad 9 \quad 10 \quad \therefore GCF = 4 \cdot 4 = 16$$

[EX15]

$$\textcircled{8} \overline{) 456 \quad 768} \\ \textcircled{3} \overline{) 57 \quad 96} \\ \quad \quad 19 \quad 32 \quad \therefore GCF = 24 \Rightarrow 24 \text{ cm by } 24 \text{ cm} \Rightarrow \frac{456 \times 768}{24 \times 24} = 608 \text{ tiles}$$

[EX16]

$$\textcircled{3} \overline{) 12 \quad 15 \quad 18} \\ \textcircled{2} \overline{) 4 \quad 5 \quad 6} \\ \quad \quad \textcircled{2} \quad \textcircled{5} \quad \textcircled{3} \quad LCM = 3 \cdot 2 \cdot 2 \cdot 5 \cdot 3 = 180 \text{ min} \\ \therefore 10:00 \text{ AM}$$

[EX17]

$$\textcircled{3} \overline{) 6 \quad 4 \quad 9} \\ \textcircled{2} \overline{) 2 \quad 4 \quad 3} \\ \quad \quad \textcircled{1} \quad \textcircled{2} \quad \textcircled{3} \quad LCM = 3 \cdot 2 \cdot 3 \cdot 2 = 36 \text{ days} \\ 1 + 36 - 31 (\text{Jan}) = 6 \\ \therefore \text{Feb } 6\text{th}$$

[EX18]

$$\textcircled{3} \overline{) 6 \quad 12 \quad 21 \quad 35} \\ \textcircled{2} \overline{) 2 \quad 4 \quad 7 \quad 35} \\ \textcircled{7} \overline{) 1 \quad 2 \quad 7 \quad 35} \\ \quad \quad \textcircled{1} \quad \textcircled{2} \quad \textcircled{1} \quad \textcircled{5} \quad LCM = 3 \cdot 2 \cdot 7 \cdot 2 \cdot 5 = 420 \Rightarrow 420, 840, 1260, 1680, \dots \\ \therefore \text{The smallest 4-digit number is } 1260.$$

★ Square Roots and Cube Roots: p22

$$[EX1] \textcircled{1} \text{ Yes} \quad \textcircled{2} \text{ Yes} \quad \textcircled{3} \text{ Yes} \quad \textcircled{4} \text{ No} \quad (2592 = 4^2 \cdot 2 \cdot 9^2) \quad \textcircled{5} \text{ Yes}$$

$$[EX2] \textcircled{1} \sqrt{121} = 11 \quad \textcircled{2} \sqrt{729} = 27 \quad \textcircled{3} \sqrt{1024} = 32 \quad \textcircled{4} \sqrt{625} = 25 \quad \textcircled{5} \sqrt{6400} = 80 \quad \textcircled{6} \sqrt{2304} = 48$$

$$\textcircled{7} \sqrt{12100} = 110 \quad \textcircled{8} \sqrt{1.44} = 1.2 \quad \textcircled{9} \sqrt{0.0169} = 0.13 \quad \textcircled{10} \sqrt{\frac{9}{16}} = \frac{3}{4} \quad \textcircled{11} \sqrt{\frac{25}{81}} = \frac{5}{9} \quad \textcircled{12} \sqrt{\frac{441}{100}} = \frac{21}{10}$$

[EX2]

$$\begin{array}{r} \textcircled{1} \quad 2x^2 - 3x - 2 \\ \begin{array}{r} \boxed{1} \rightarrow -2 \\ \boxed{2} \rightarrow 1 \end{array} \\ \hline -3 \end{array}$$

$$= (x-2)(2x+1)$$

$$\begin{array}{r} \textcircled{4} \quad 3x^2 + 5x - 2 \\ \begin{array}{r} \boxed{1} \rightarrow 2 \\ \boxed{3} \rightarrow -1 \end{array} \\ \hline 5 \end{array}$$

$$= (x+2)(3x-1)$$

$$\begin{array}{r} \textcircled{7} \quad -(4x^2 + x - 3) \\ \begin{array}{r} \boxed{1} \rightarrow 1 \\ \boxed{4} \rightarrow -3 \end{array} \\ \hline 1 \end{array}$$

$$= -(x+1)(4x-3)$$

$$\begin{array}{r} \textcircled{10} \quad -(3a^2 - 10a - 8) \\ \begin{array}{r} \boxed{3} \rightarrow 2 \\ \boxed{1} \rightarrow -4 \end{array} \\ \hline -10 \end{array}$$

$$= -(3a+2)(a-4)$$

$$\begin{array}{r} \textcircled{13} \quad 25x^2 + 40xy + 16y^2 \\ \begin{array}{r} \boxed{5} \rightarrow 4 \\ \boxed{5} \rightarrow 4 \end{array} \\ \hline 40 \end{array}$$

$$= (5x+4y)^2$$

$$\textcircled{16} = (3a-5b)(3a+b)$$

$$\textcircled{19} = (2x-3y)(3x-2y)$$

$$\begin{aligned} \textcircled{22} &= -(4s^2 - 13st + 10t^2) \\ &= -(4s-5t)(s-2t) \end{aligned}$$

$$\textcircled{25} = (7p-4q)(p+2q)$$

$$\begin{array}{r} \textcircled{2} \quad 2x^2 + 3x - 5 \\ \begin{array}{r} \boxed{1} \rightarrow -1 \\ \boxed{2} \rightarrow 5 \end{array} \\ \hline 3 \end{array}$$

$$= (x-1)(2x+5)$$

$$\begin{array}{r} \textcircled{5} \quad 3x^2 - x - 2 \\ \begin{array}{r} \boxed{1} \rightarrow -1 \\ \boxed{3} \rightarrow 2 \end{array} \\ \hline -1 \end{array}$$

$$= (x-1)(3x+2)$$

$$\begin{array}{r} \textcircled{8} \quad 4y^2 + 4y - 3 \\ \begin{array}{r} \boxed{2} \rightarrow -1 \\ \boxed{2} \rightarrow 3 \end{array} \\ \hline 4 \end{array}$$

$$= (2y-1)(2y+3)$$

$$\begin{array}{r} \textcircled{11} \quad -(6x^2 + x - 1) \\ \begin{array}{r} \boxed{2} \rightarrow 1 \\ \boxed{3} \rightarrow -1 \end{array} \\ \hline 1 \end{array}$$

$$= -(2x+1)(3x-1)$$

$$\begin{array}{r} \textcircled{14} \quad 6m^2 + 11mn - 10n^2 \\ \begin{array}{r} \boxed{2} \rightarrow 5 \\ \boxed{3} \rightarrow -2 \end{array} \\ \hline 11 \end{array}$$

$$= (2m+5n)(3m-2n)$$

$$\textcircled{17} = (2p+3q)(3p-2q)$$

$$\textcircled{20} = (3x-7y)^2$$

$$\begin{aligned} \textcircled{23} &= -(8x^2 - 13xy - 6y^2) \\ &= -(8x+3y)(x-2y) \end{aligned}$$

$$\textcircled{26} = (7x+3y)(x-2y)$$

$$\begin{array}{r} \textcircled{3} \quad 2a^2 - a - 6 \\ \begin{array}{r} \boxed{2} \rightarrow 3 \\ \boxed{1} \rightarrow -2 \end{array} \\ \hline -1 \end{array}$$

$$= (2a+3)(a-2)$$

$$\begin{array}{r} \textcircled{6} \quad 3m^2 - 2m - 5 \\ \begin{array}{r} \boxed{1} \rightarrow 1 \\ \boxed{3} \rightarrow -5 \end{array} \\ \hline -2 \end{array}$$

$$= (m+1)(3m-5)$$

$$\begin{array}{r} \textcircled{9} \quad -(5t^2 - 3t - 8) \\ \begin{array}{r} \boxed{1} \rightarrow 1 \\ \boxed{5} \rightarrow -8 \end{array} \\ \hline -3 \end{array}$$

$$= -(t+1)(5t-8)$$

$$\begin{array}{r} \textcircled{12} \quad -(2x^2 - 13x - 7) \\ \begin{array}{r} \boxed{2} \rightarrow 1 \\ \boxed{1} \rightarrow -7 \end{array} \\ \hline -13 \end{array}$$

$$= -(2x+1)(x-7)$$

$$\begin{array}{r} \textcircled{15} \quad 9a^2 - 4ab - 5b^2 \\ \begin{array}{r} \boxed{9} \rightarrow 5 \\ \boxed{1} \rightarrow -1 \end{array} \\ \hline -4 \end{array}$$

$$= (9a+5b)(a-b)$$

$$\textcircled{18} = -(9x^2 + 9xy - 4y^2) = -(3x-y)(3x+4y)$$

$$\textcircled{21} = (2m+n)(3m-4n)$$

$$\begin{aligned} \textcircled{24} &= -(7a^2 + 17ab - 12b^2) \\ &= -(a+3b)(7a-4b) \end{aligned}$$

$$\textcircled{27} = (3m+2n)(3m-4n)$$

[EX3]

$$\textcircled{1} = (x^2+4)(5x^2-6)$$

$$\textcircled{4} = (2x+3y)(6x-5y)$$

$$\textcircled{2} = (ab-6)(3ab+8)$$

$$\textcircled{5} = (4p-5q)(8p+3q)$$

$$\textcircled{3} = (t^3-15)(2t^3-5)$$

$$\textcircled{6} = -(24a^2 - 41ab + 12b^2) = -(3a-4b)(8a-3b)$$

[EX4]

$$\textcircled{1} = 2(x+2)(7x-4)$$

$$\begin{aligned} \textcircled{4} &= 2x(4x^2 - 20x + 25) \\ &= 2x(2x-5)^2 \end{aligned}$$

$$\begin{aligned} \textcircled{7} &= 5x(9x^2 + 12x + 4) \\ &= 5x(3x+2)^2 \end{aligned}$$

$$\textcircled{2} = 5(x-2y)(7x+3y)$$

$$\begin{aligned} \textcircled{5} &= 4(10m^2 + 13mn - 3n^2) \\ &= 4(2m+3n)(5m-n) \end{aligned}$$

$$\begin{aligned} \textcircled{8} &= -2a(10a^2 - 9ab - 36b^2) \\ &= -2a(2a+3b)(5a-12b) \end{aligned}$$

$$\textcircled{3} = -3(3m+2n)(3m-4n)$$

$$\begin{aligned} \textcircled{6} &= -x^2(12x^2 + 13xy - 14y^2) \\ &= -x^2(3x-2y)(4x+7y) \end{aligned}$$

$$\begin{aligned} \textcircled{9} &= ab(21a^2 + 13ab - 20b^2) \\ &= ab(3a+4b)(7a-5b) \end{aligned}$$

[EX5]

$$\textcircled{1} \text{ Let } A = 2x+3y.$$

$$\begin{array}{r} 3A^2 - A - 2 \\ \begin{array}{r} \boxed{3} \rightarrow 2 \\ \boxed{1} \rightarrow -1 \end{array} \\ \hline -1 \end{array}$$

$$\begin{aligned} &= [3A+2][A-1] \\ &= [3(2x+3y)+2][(2x+3y)-1] \\ &= (6x+9y+2)(2x+3y-1) \end{aligned}$$

$$\textcircled{2} \text{ Let } A = x^2+2x+2.$$

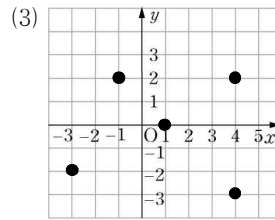
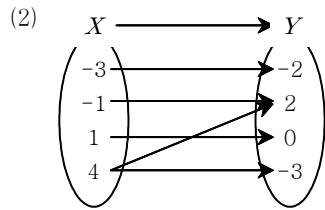
$$\begin{array}{r} A^2 - 6A + 5 \\ \begin{array}{r} \boxed{1} \rightarrow -1 \\ \boxed{1} \rightarrow -5 \end{array} \\ \hline -6 \end{array}$$

$$\begin{aligned} &= [A-1][A-5] \\ &= [(x^2+2x+2)-1][(x^2+2x+2)-5] \\ &= (x^2+2x+1)(x^2+2x-3) \\ &= (x+1)^2(x+3)(x-1) \end{aligned}$$

Chapter Review Exercises: p154

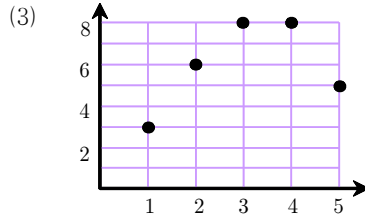
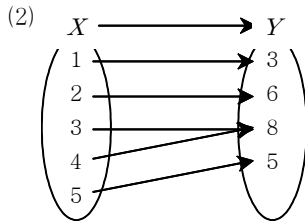
1. ① (1)

| x | y |
|-----|-----|
| -3 | -2 |
| -1 | 2 |
| 1 | 0 |
| 4 | 2 |
| 4 | -3 |



② $D: \{-3, -1, 1, 4\}$
 $R: \{-3, -2, 0, 2\}$

2. (1) $\{(1, 3), (2, 6), (3, 8), (4, 8), (5, 5)\}$



3. (1) $\{(-2, 3), (-1, 0), (0, 3), (2, -1), (3, 4), (5, -3)\}$

(2) $D: \{-2, -1, 0, 2, 3, 5\}$
 $R: \{-3, -1, 0, 3, 4\}$

4. ① (1) $D: \{x \mid -3 \leq x \leq 5, x \in \mathbb{R}\}$
 $R: \{y \mid -1 \leq y \leq 3, y \in \mathbb{R}\}$

② (1) $D: \{x \mid -3 < x < 4, x \in \mathbb{R}\}$
 $R: \{y \mid -2 \leq y < 3, y \in \mathbb{R}\}$

③ (1) $D: \{x \mid x \leq 5, x \in \mathbb{R}\}$
 $R: \{y \mid -2 \leq y < 0 \text{ or } y \geq 2, y \in \mathbb{R}\}$

(2) $D: [-3, 5], R: [-1, 3]$

(2) $D: (-3, 4), R: [-2, 3]$

(2) $D: (-\infty, 5], R: [-2, 0) \cup [2, \infty)$

5. ① (1) $D: \{x \mid x \geq -1, x \in \mathbb{R}\}$
 $R: \{y \mid y \in \mathbb{R}\}$

② (1) $D: \{x \mid x \in \mathbb{R}\}$
 $R: \{y \mid -2 < y < 3, y \in \mathbb{R}\}$

③ (1) $D: \{x \mid -3 \leq x < 3, x \in \mathbb{R}\}$
 $R: \{y \mid y = -2, 0, 3\}$

④ (1) $D: \{x \mid x < 0 \text{ or } x \geq 2, x \in \mathbb{R}\}$
 $R: \{y \mid y \in \mathbb{R}\}$

(2) Not a function.

(2) Function; 1-to-1.

(2) Function; Not 1-to-1.

(2) Function; 1-to-1.

6. ① $= 2[6(3) - 5]$
 $= 2(18 - 5) = 26$

② $= -[6(x+2) - 5]$
 $= -[6x + 12 - 5]$
 $= -(6x + 7) = -6x - 7$

③ $= \frac{[6(x+h) - 5] - [6x - 5]}{h}$
 $= \frac{6x + 6h - 5 - 6x + 5}{h} = \frac{6h}{h} = 6$

④ $6x - 5 = 19$
 $6x = 24$
 $x = 4$

7. ① $= (-2)^2 - 3(-2) + 4$
 $= 4 + 6 + 4$
 $= 14$

② $= (x-2)^2 - 3(x-2) + 4$
 $= x^2 - 4x + 4 - 3x + 6 + 4$
 $= x^2 - 7x + 14$

③ $= \frac{(x+h)^2 - 3(x+h) + 4 - [x^2 - 3x + 4]}{h}$
 $= \frac{x^2 + 2xh + h^2 - 3x - 3h + 4 - x^2 + 3x - 4}{h}$
 $= \frac{2xh + h^2 - 3h}{h} = 2x + h - 3$

④ $x^2 - 3x + 4 = 4$
 $x^2 - 3x = 0$
 $x(x-3) = 0$
 $x = 0, 3$

8. ① $= 4$

② $= 0$

③ $= -2$

④ $x = -3, 3, 5$

⑤ $x = -2, 2$

⑥ $-1 \leq x \leq 1$

⑦ $-2 < x < 2$

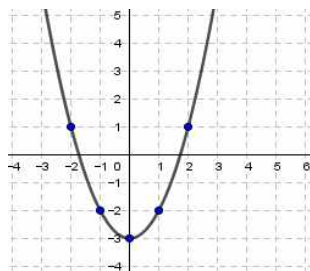
⑧ $= f(-2) - (-2) = 2 + 2 = 4$

⑨ $= f(2) - 2 = 2 - 2 = 0$

⑩ $= \frac{0-4}{4} = -1$

9. ① (1)

| x | y |
|-----|-----|
| -2 | 1 |
| -1 | -2 |
| 0 | -3 |
| 1 | -2 |
| 2 | 1 |

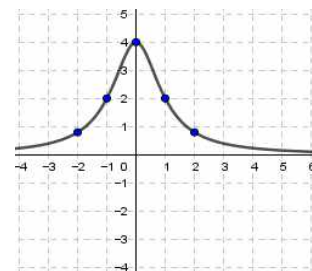


(2) Function

(3) $D: \{x \mid x \in \mathbb{R}\}$
 $R: \{y \mid y \geq -3, y \in \mathbb{R}\}$

② (1)

| x | y |
|-----|-------|
| -2 | $4/5$ |
| -1 | 2 |
| 0 | 4 |
| 1 | 2 |
| 2 | $4/5$ |

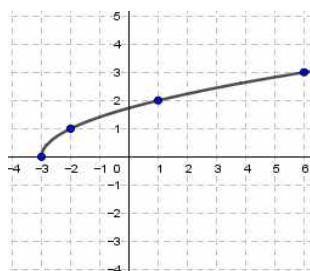


(2) Function

(3) $D: \{x \mid x \in \mathbb{R}\}$
 $R: \{y \mid 0 < y \leq 4, y \in \mathbb{R}\}$

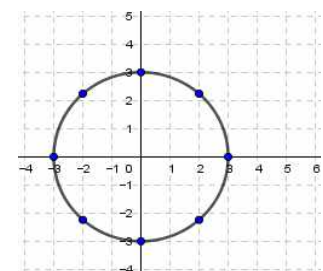
③ (1)

| x | y |
|-----|-----|
| -3 | 0 |
| -2 | 1 |
| 1 | 2 |
| 6 | 3 |

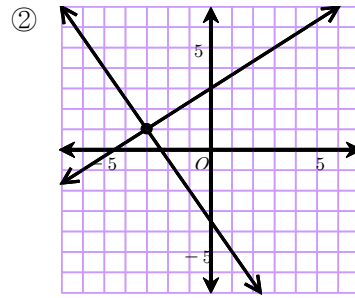
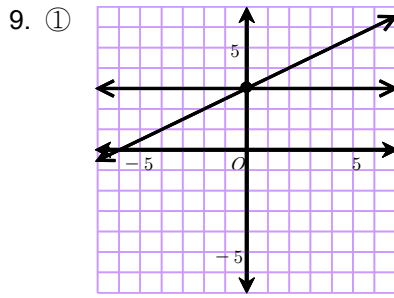


④ (1)

| x | y |
|-----|--------------------------------|
| -3 | 0 |
| -2 | $\pm\sqrt{5} \approx \pm 2.24$ |
| 0 | ± 3 |
| 2 | $\pm\sqrt{5} \approx \pm 2.24$ |
| 3 | 0 |



8. (1) $\frac{4}{3}$ (2) 0 (3) $\frac{1}{3}$ (4) $-\frac{1}{3}$ (5) -3 (6) undefined



10. $\frac{a+5}{3-a} = 3$
 $a+5 = 3(3-a)$
 $a+5 = 9-3a$
 $4a = 4 \therefore a = 1$

11. Let the point be $(x, 0)$.
 $\frac{0+4}{x-3} = \frac{1}{2}$
 $x-3 = 8$
 $x = 11 \therefore (11, 0)$

12. Slope of AB = Slope of BC

$$\frac{2-5}{1+3} = \frac{8-2}{k-1} \Rightarrow -3(k-1) = 24$$

$$\frac{-3}{4} = \frac{6}{k-1} \Rightarrow k-1 = -8$$

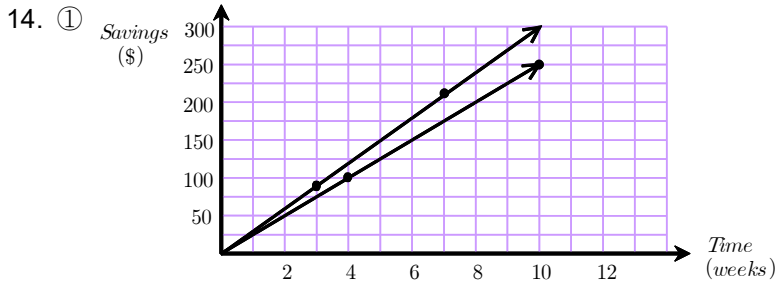
$$k = -7$$

13. (1) $(0, 2000), (6, 920)$

$$\frac{\Delta y}{\Delta x} = \frac{920-2000}{0-6} = \frac{-1080}{6} = -\$180/\text{yr}$$

(2) Change in value = $-180 \cdot 3 = -540$

$$920 - 540 = \$380$$



② Sasha: $m = \frac{210-90}{7-3} = \frac{120}{4} = \$30/\text{week}$

Benson: $m = \frac{250-100}{10-4} = \frac{150}{6} = \$25/\text{week}$

③ Sasha = $\$30/\text{week} \cdot 40\text{weeks} = \1200

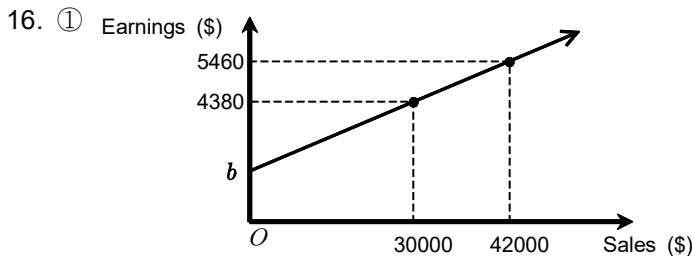
Benson = $\$25/\text{week} \cdot 40\text{weeks} = \1000

\therefore Sasha saves \$200 more than Benson.

15. $32 + 4x \leq 186$

$$4x \leq 154$$

$$x \leq 38.5 \therefore 38 \text{ packs}$$



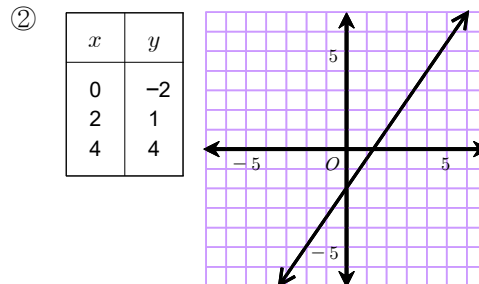
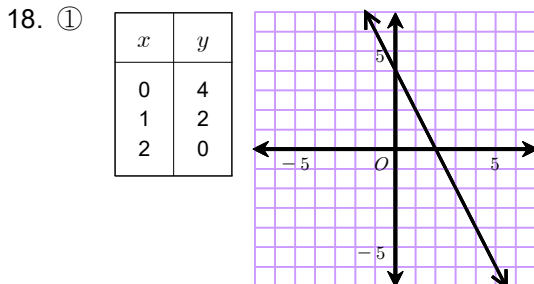
② $\frac{5460-4380}{42000-30000} = \frac{1080}{12000} = 0.09 \therefore 9\%$

③ $(0, b), (30000, 4380)$
 $\frac{4380-b}{30000-0} = 0.09$
 $4380 - b = 2700$
 $b = 4380 - 2700 = \$1680$

17. ① $\frac{5-2}{k+2} = \frac{-1+3}{7-4}$
 $\frac{3}{k+2} = \frac{2}{3} \Rightarrow 2k+4 = 9$
 $2k = 5$
 $k = \frac{5}{2}$

② $\frac{5-2}{k+2} \cdot \frac{-1+3}{7-4} = -1$
 $\frac{3}{k+2} \cdot \frac{2}{3} = -1 \Rightarrow k+2 = -2$
 $k = -4$

③ Let the point P be $(0, y)$.
 $\frac{y-2}{0+2} \cdot \frac{2}{3} = -1 \Rightarrow y-2 = -3$
 $\frac{y-2}{3} = -1 \Rightarrow y = -1$
 $\therefore (0, -1)$



19. ① $3(k-4) - 4(-3) = 24$

$$3k + 12 = 24$$

$$3k = 12$$

$$\therefore k = 4$$

② $y+3 = \frac{1}{2}(x-k), (0, 3)$

$$3+3 = \frac{1}{2}(0-k)$$

$$6 = -\frac{1}{2}k \therefore k = -12$$

③ $kx + 0 - 10 = 0$ $0 + 4y + 12 = 0$

$$x = \frac{10}{k} \quad y = -3$$

$$\frac{10}{k} = -3 \Rightarrow \therefore k = -\frac{10}{3}$$

[EX9] ① $-5y = -2x - 10$

$$y = \frac{2}{5}x + 2 \quad \therefore m_2 = \frac{2}{5}$$

Let $y = \frac{2}{5}x + b$. $(5, -2)$

$$-2 = \frac{2}{5}(5) + b$$

$$-2 = 2 + b, \quad b = -4$$

$$\therefore y = \frac{2}{5}x - 4$$

② $9y = -3x + 15$

$$y = -\frac{1}{3}x + \frac{5}{3} \quad \therefore m_2 = 3$$

Let $y = 3x + b$. $(-3, 4)$

$$4 = -9 + b$$

$$b = 13$$

$$\therefore y = 3x + 13$$

③ $-4y = -3x + 12$

$$y = \frac{3}{4}x - 3 \quad \therefore m_2 = -\frac{4}{3}$$

$$3(0) - 4y = 12$$

$$y = -3$$

$$\therefore y\text{-int} = -3$$

$$\therefore y = -\frac{4}{3}x - 3$$

④ $m_1 = \frac{5-8}{2+4} = \frac{-3}{6} = -\frac{1}{2}$

$$\therefore m_2 = -\frac{1}{2}$$

Let $y = -\frac{1}{2}x + b$. $(5, 0)$

$$0 = -\frac{5}{2} + b, \quad b = \frac{5}{2}$$

$$\therefore y = -\frac{1}{2}x + \frac{5}{2}$$

[EX10] ① $m_1 = 4, \quad m_2 = -\frac{1}{4}$

\therefore perpendicular

② $m_1 = \frac{9}{6} = \frac{3}{2}, \quad m_2 = \frac{3}{2}$

\therefore parallel

③ $m_1 = \frac{4}{5}, \quad m_2 = \frac{10}{8} = \frac{5}{4}$

\therefore neither

④ $m_1 = -\frac{7}{3}, \quad m_2 = \frac{3}{7}$

\therefore perpendicular

[EX11] ① $\frac{k-6}{2-k} = \frac{1}{3}$

$$3k - 18 = 2 - k$$

$$4k = 20, \quad \therefore k = 5$$

② $6y = -4x + 12$

$$y = -\frac{2}{3}x + 2 \quad \Rightarrow$$

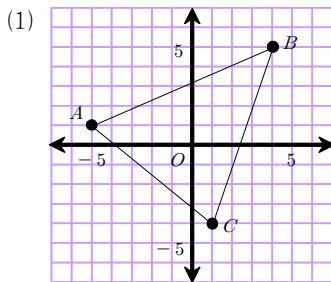
$$\therefore m_2 = \frac{3}{2}$$

$$\frac{k+1}{14-k} = \frac{3}{2}$$

$$2k + 2 = 42 - 3k$$

$$5k = 40, \quad \therefore k = 8$$

[EX12]

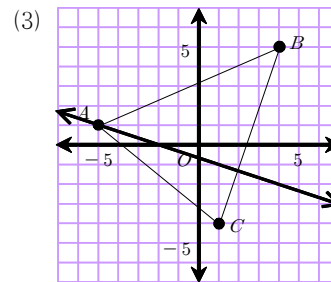


(2) $m_1 = \frac{-4-5}{1-4} = \frac{-9}{-3} = 3$

Let $y = 3x + b$. $(4, 5)$

$$5 = 12 + b, \quad b = -7$$

$$\therefore y = 3x - 7$$



(4) $m_2 = -\frac{1}{3}$

Let $y = -\frac{1}{3}x + b$. $(-5, 1)$

$$1 = \frac{5}{3} + b, \quad b = -\frac{2}{3}$$

$$\therefore y = -\frac{1}{3}x - \frac{2}{3}$$

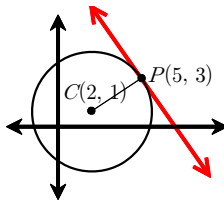
[EX13] Radius \perp Tangent line.

$$\text{Slope of CP} = \frac{3-1}{5-2} = \frac{2}{3}$$

$$\therefore \text{Slope of the tangent line} = -\frac{3}{2}$$

Let $y = -\frac{3}{2}x + b$. $(5, 3)$

$$3 = -\frac{15}{2} + b, \quad b = \frac{21}{2} \quad \Rightarrow \quad \therefore y = -\frac{3}{2}x + \frac{21}{2}$$



[EX14] $x - 2y + 4 = 0$ $bx + y + c = 0$

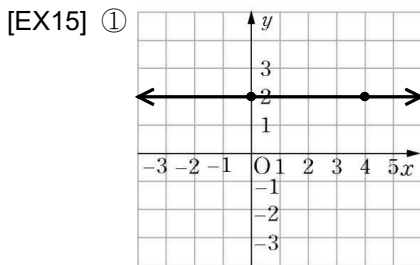
$$\Rightarrow y = \frac{1}{2}x + 2 \quad \Rightarrow y = -bx - c$$

$$\frac{1}{2} \times (-b) = -1$$

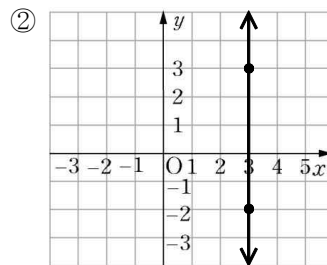
$$\therefore b = 2$$

The two lines have the same y -intercept.

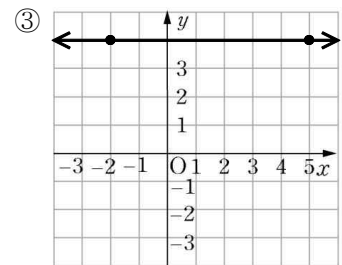
$$2 = -c \quad \therefore c = -2$$



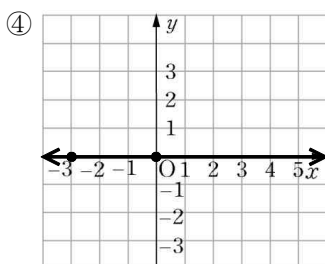
$$y = 2$$



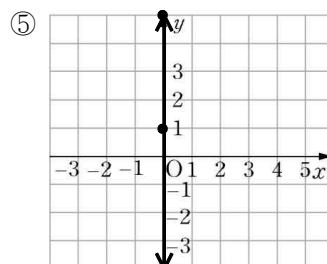
$$x = 3$$



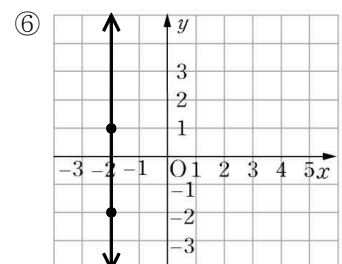
$$y = 4$$



$$y = 0$$

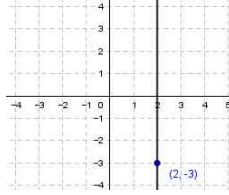


$$x = 0$$



$$x = -2$$

$$\begin{array}{l}
 10. \textcircled{1} \quad \begin{cases} 4x + 2y - 2 = 0 \\ 3x - 2y - 12 = 0 \end{cases} \\
 \quad \quad \quad \begin{array}{r} 7x \quad -14 = 0 \\ x = 2 \end{array} \\
 \quad \quad \quad \begin{array}{l} 2(2) + y - 1 = 0 \\ y = -3 \\ \Rightarrow P(2, -3) \end{array}
 \end{array}$$



$\therefore x = 2$

$$\begin{aligned}
 \textcircled{2} \quad m_1 = \frac{5}{4} &\Rightarrow m_2 = -\frac{4}{5} \\
 \therefore y + 3 &= -\frac{4}{5}(x - 2) \\
 \text{(or } y &= -\frac{4}{5}x - \frac{7}{5})
 \end{aligned}$$

$$\begin{aligned}
 11. \text{ Step 1. } m_1 = \frac{4}{3} &\Rightarrow m_2 = -\frac{3}{4} \\
 \text{perpendicular line through } (5, -1) &: y + 1 = -\frac{3}{4}(x - 5) \\
 y &= -\frac{3}{4}x + \frac{15}{4} - 1 \\
 y &= -\frac{3}{4}x + \frac{11}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{Step 2. Intersection point: } \frac{4}{3}x + \frac{2}{3} &= -\frac{3}{4}x + \frac{11}{4} \\
 \text{Multiply by 12. } 16x + 8 &= -9x + 33 \\
 25x &= 25 \\
 x = 1 &\rightarrow y = \frac{4}{3}(1) + \frac{2}{3} = \frac{6}{3} = 2 \\
 \therefore H(1, 2)
 \end{aligned}$$

$$\text{Step 3. Distance PH} = \sqrt{(1-5)^2 + (2+1)^2} = \sqrt{16+9} = 5$$

$$\begin{aligned}
 12. \text{ Let the two numbers be } x \text{ and } y. (x > y) \\
 (1) x = y + 8 \quad (1) \rightarrow (2): (y + 8) + 3y &= -12 \\
 (2) x + 2y = \frac{1}{3}x - 8 \quad 4y &= -20 \\
 3x + 6y = x - 24 \quad y &= -5, \quad x = -5 + 8 = 3 \\
 2x + 6y = -24 \quad \therefore & 3, -5 \\
 x + 3y = -12
 \end{aligned}$$

$$\begin{aligned}
 13. \text{ Let } x \text{ and } y \text{ be the tens digit and the ones digit, respectively.} \\
 (1) y = 2x - 1 \quad (1) \rightarrow (2): -x + (2x - 1) &= 2 \\
 (2) 10y + x = 10x + y + 18 \quad x - 1 &= 2 \\
 -9x + 9y = 18 \quad x &= 3 \\
 -x + y = 2 \quad y = 2(3) - 1 &= 5 \\
 \therefore 35
 \end{aligned}$$

14. Let x be the number of adult tickets and y be the number of student tickets.

$$\begin{array}{l}
 (1) x + y = 120 \\
 (2) 30x + 18y = 3060 \\
 5x + 3y = 510
 \end{array}
 \quad - \quad
 \begin{array}{r}
 5x + 3y = 510 \\
 3x + 3y = 360 \\
 \hline
 2x = 150 \\
 x = 75, y = 45
 \end{array}
 \quad \therefore 75 \text{ adult tickets and } 45 \text{ student tickets.}$$

15. Let x be the speed of the motorboat and y be the speed of the current.

| | D | S | T |
|------|----|---------|---|
| Up | 60 | $x - y$ | 4 |
| Down | 60 | $x + y$ | 3 |

$$\begin{aligned}
 (1) 4(x - y) &= 60 \\
 x - y &= 15 \\
 (2) 3(x + y) &= 60 \\
 x + y &= 20 \\
 \therefore \text{The speed of the motorboat: } 17.5 \text{ km/h} \\
 \text{The speed of the current: } 2.5 \text{ km/h}
 \end{aligned}$$

$$\begin{array}{r}
 x - y = 15 \\
 + \quad x + y = 20 \\
 \hline
 2x = 35 \\
 x = 17.5, y = 2.5
 \end{array}$$

16. Let x be the time spent travelling by car and y be the time spent travelling by train.

| | D | S | T |
|-------|--------|-----|-----|
| Car | $100x$ | 100 | x |
| Train | $80y$ | 80 | y |

$$\begin{aligned}
 (1) x + y &= 5.5 \\
 (2) 100x + 80y &= 520 \\
 5x + 4y &= 26 \\
 \therefore 4 \text{ h by car and } 1.5 \text{ h by train.}
 \end{aligned}$$

$$\begin{array}{r}
 5x + 4y = 26 \\
 - \quad 4x + 4y = 22 \\
 \hline
 x = 4, y = 1.5
 \end{array}$$

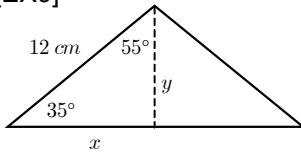
17. Let x be the number of mL of 50% solution used and y be the number of mL of 95% solution used.

$$\begin{aligned}
 (1) x + y &= 900 \\
 (2) 0.5x + 0.95y &= 0.7 \cdot 900 \\
 0.5x + 0.95y &= 630 \\
 x + 1.9y &= 1260 \\
 \therefore 500 \text{ mL of the } 50\% \text{ solution and } 400 \text{ mL of the } 95\% \text{ solution.}
 \end{aligned}$$

$$\begin{array}{r}
 x + 1.9y = 1260 \\
 - \quad x + y = 900 \\
 \hline
 0.9y = 360 \\
 y = 400, x = 500
 \end{array}$$

$$\begin{aligned}
 18. \quad \textcircled{1} + \textcircled{2}: 2x + y &= 3 \quad \textcircled{4} \\
 \textcircled{2} \times 2 + \textcircled{3}: 5x - y &= 4 \quad \textcircled{5} \\
 \textcircled{4} + \textcircled{5}: 7x &= 7 \\
 \therefore x &= 1 \\
 \textcircled{4}: 2(1) + y &= 3 \quad \textcircled{4} \\
 \therefore y &= 1 \\
 \textcircled{1}: (1) + 2(1) + z &= 5 \\
 \therefore z &= 2 \\
 \therefore (1, 1, 2)
 \end{aligned}$$

[EX9]



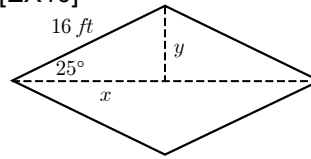
$$x = 12 \cos 35^\circ = 9.829 \dots$$

$$y = 12 \sin 35^\circ = 6.882 \dots$$

$$A = \frac{1}{2} (9.829 \dots) (6.882 \dots) \times 2$$

$$= 67.65 \dots = 67.7 \text{ cm}^2$$

[EX10]



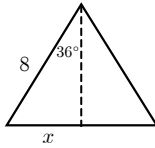
$$x = 16 \cos 25^\circ = 14.50 \dots$$

$$y = 16 \sin 25^\circ = 6.76 \dots$$

$$A = \frac{1}{2} (14.50 \dots) (6.76 \dots) \times 4$$

$$= 196.10 \dots = 196.1 \text{ ft}^2$$

[EX11] ①



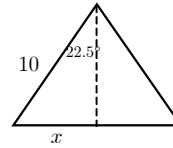
$$\sin 36^\circ = \frac{x}{8}$$

$$x = 8 \sin 36^\circ = 4.70 \dots$$

$$\text{perimeter} = 10 \cdot x = 47.02 \dots$$

$$= 47.0 \text{ cm}$$

②



$$\sin 22.5^\circ = \frac{x}{10}$$

$$x = 10 \sin 22.5^\circ = 3.82 \dots$$

$$\text{perimeter} = 16 \cdot x = 61.22 \dots$$

$$= 61.2 \text{ in}$$

[EX12]

$$\tan 22^\circ = \frac{AH}{x} \quad \tan 42^\circ = \frac{HB}{x}$$

$$AH = x \tan 22^\circ \quad HB = x \tan 42^\circ$$

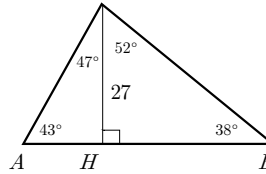
$$AH + HB = 200$$

$$x \tan 22^\circ + x \tan 42^\circ = 200$$

$$x (\tan 22^\circ + \tan 42^\circ) = 200$$

$$x = \frac{200}{\tan 22^\circ + \tan 42^\circ} = 153.32 \dots = 153.3 \text{ m}$$

[EX13]



$$\tan 47^\circ = \frac{AH}{27} \quad \tan 52^\circ = \frac{HB}{27}$$

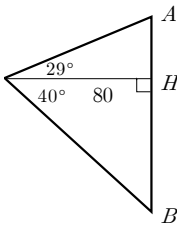
$$AH = 27 \tan 47^\circ \quad HB = 27 \tan 52^\circ$$

$$AB = AH + HB$$

$$AB = 27 \tan 47^\circ + 27 \tan 52^\circ$$

$$= 63.51 \dots = 63.5 \text{ m}$$

[EX14]



$$\tan 29^\circ = \frac{AH}{80} \quad \tan 40^\circ = \frac{HB}{80}$$

$$AH = 80 \tan 29^\circ \quad HB = 80 \tan 40^\circ$$

$$AB = AH + HB$$

$$= 80 \tan 29^\circ + 80 \tan 40^\circ$$

$$= 111.47 \dots = 111.5 \text{ m}$$

[EX15]

$$\angle ACD = 41^\circ \quad \tan 41^\circ = \frac{AD}{x} \quad AD = x \tan 41^\circ$$

$$\angle BCD = 35^\circ \quad \tan 35^\circ = \frac{BD}{x} \quad BD = x \tan 35^\circ$$

$$AD - BD = 30$$

$$x \tan 41^\circ - x \tan 35^\circ = 30$$

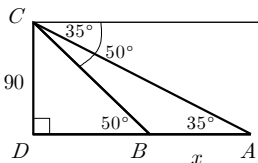
$$x (\tan 41^\circ - \tan 35^\circ) = 30$$

$$x = \frac{30}{\tan 41^\circ - \tan 35^\circ}$$

$$= 177.43 \dots$$

$$= 177.4 \text{ m}$$

[EX16]



$$\angle ACD = 55^\circ \quad \tan 55^\circ = \frac{AD}{90} \quad AD = 90 \tan 55^\circ$$

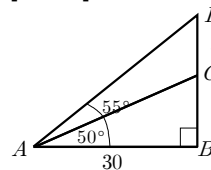
$$\angle BCD = 40^\circ \quad \tan 40^\circ = \frac{BD}{90} \quad BD = 90 \tan 40^\circ$$

$$AB = AD - BD$$

$$= 90 \tan 55^\circ - 90 \tan 40^\circ$$

$$= 53.01 \dots = 53.0 \text{ ft}$$

[EX17]



$$\tan 55^\circ = \frac{BD}{30} \quad \tan 50^\circ = \frac{BC}{30}$$

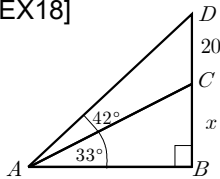
$$BD = 30 \tan 55^\circ \quad BC = 30 \tan 50^\circ$$

$$x = BD - BC$$

$$= 30 \tan 55^\circ - 30 \tan 50^\circ$$

$$= 7.09 \dots = 7.1 \text{ m}$$

[EX18]



$$\angle ADB = 48^\circ \quad \angle ACB = 57^\circ$$

$$\tan 48^\circ = \frac{AB}{20+x} \quad \tan 57^\circ = \frac{AB}{x}$$

$$AB = (20+x) \tan 48^\circ \quad (1) \quad AB = x \tan 57^\circ \quad (2)$$

$$(20+x) \tan 48^\circ = x \tan 57^\circ$$

$$20 \tan 48^\circ + x \tan 48^\circ = x \tan 57^\circ$$

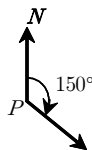
$$20 \tan 48^\circ = x \tan 57^\circ - x \tan 48^\circ$$

$$20 \tan 48^\circ = x (\tan 57^\circ - \tan 48^\circ)$$

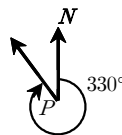
$$x = \frac{20 \tan 48^\circ}{\tan 57^\circ - \tan 48^\circ}$$

$$= 51.74 \dots = 51.7 \text{ ft}$$

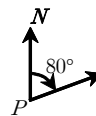
[EX19] ①



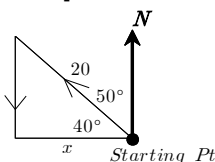
②



③



[EX20]



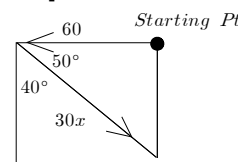
$$10 \text{ mph} \cdot 2 \text{ h} = 20 \text{ miles}$$

$$\cos 40^\circ = \frac{x}{20}$$

$$x = 20 \cos 40^\circ$$

$$= 15.32 \dots = 15.3 \text{ miles}$$

[EX21]



$$20 \text{ km/h} \cdot 3 \text{ h} = 60 \text{ km}$$

$$30 \text{ km/h} \cdot x \text{ h} = 30x \text{ km}$$

$$\cos 50^\circ = \frac{60}{30x} = \frac{2}{x}$$

$$x = \frac{2}{\cos 50^\circ} = 3.11 \dots = 3.1 \text{ hr}$$

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