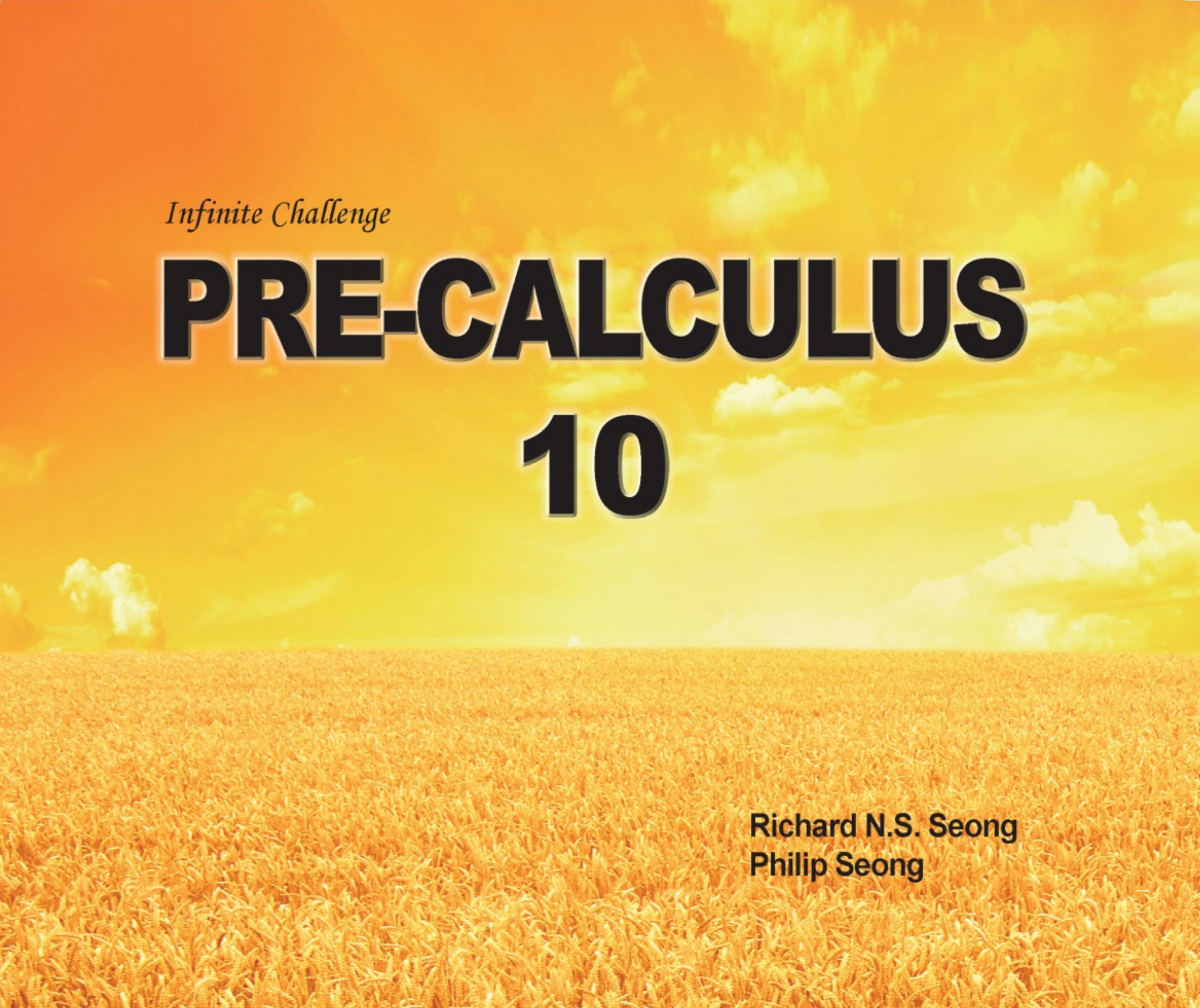


Infinite Challenge

PRE-CALCULUS 10

Richard N.S. Seong
Philip Seong

- ✔ Key points concisely summarized
- ✔ More than 400 complete guided examples
- ✔ More than 3300 carefully crafted questions with full solutions!
- ✔ 1200 graphs and diagrams to help explain and solve problems!

Infinite Challenge Publishing Ltd.

Table of Contents

Chapter 6. Systems of Linear Equations

Chapter 7. Arithmetic Sequences and Series

Chapter 8. Right Triangle Trigonometry

Chapter 9. Financial Literacy

1. Real Numbers

★ Real Number System

◆ Real Number System

- **Real Numbers**
  - **Rational Numbers**
    - **Integers** $0,\ \pm 1,\ \pm 2,\ \cdots$
      - **Whole Numbers** $0,\ 1,\ 2,\ 3,\ \cdots$
        - **Natural Numbers** $1,\ 2,\ 3,\ 4,\ \cdots$
        - **Zero** 0
      - **Negative Integers** $-1,\ -2,\ -3,\ \cdots$
    - **Non-Integers** (repeating *or* terminating decimals)
      $\frac{1}{3}=0.\bar{3}$, $\frac{38}{11}=3.\overline{45}$, $\frac{12}{5}=2.4$, $\frac{1}{4}=0.25$, $\cdots$
  - **Irrational Numbers** (non-repeating *and* non-terminating decimals)
    π, $\sqrt{2}$, $\sqrt{3}$, $-\sqrt[3]{7}$, $\cdots$

- **Rational Numbers ($\mathbb{Q}$):** Any real numbers that can be written **in the form $\frac{p}{q}$.** ($p\in\mathbb{Z}$, $q\in\mathbb{Z}$ and $q\neq 0$)

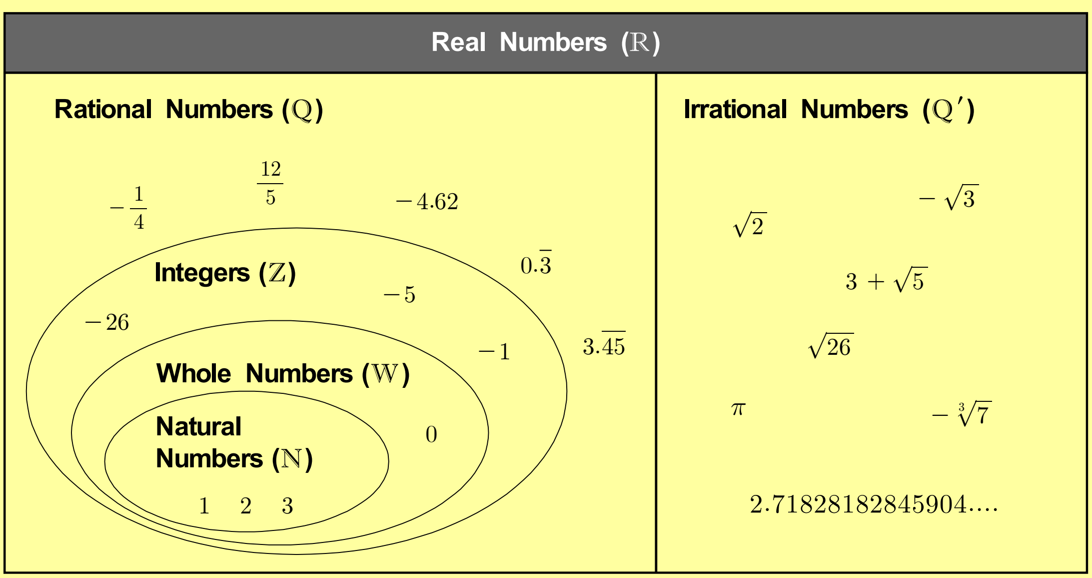
  (e.g.) $-0.7=-\frac{7}{10}$, $0.\bar{3}=\frac{1}{3}$, $6=\frac{6}{1}$, $-5=\frac{-10}{2}$

- **Irrational Numbers ($\mathbb{Q}'$):** Any real numbers that cannot be written in the form $\frac{p}{q}$.

  Irrational numbers are non-repeating *and* non-terminating decimals.

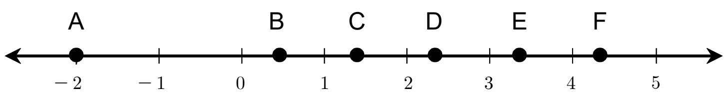
  (e.g.) $\pi=3.141592653....$, $\sqrt{2}=1.414213562....$, $\sqrt{3}=1.732050807....$

- **Integers ($\mathbb{Z}$):** $\{0,\ \pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ ......\}$
- **Whole Numbers ($\mathbb{W}$):** $\{0,\ 1,\ 2,\ 3,\ 4,\ ......\}$
- **Natural Numbers ($\mathbb{N}$):** $\{1,\ 2,\ 3,\ 4,\ ......\}$

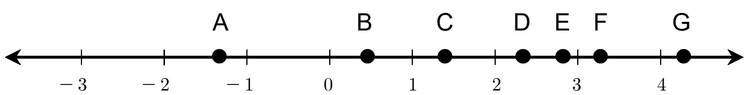


[EX12] Match each number with its corresponding point on the number line. Then arrange the numbers in order from least to greatest. Do not use a calculator.

① $\sqrt{6}$, $\sqrt[3]{-8}$, $\sqrt{20}$, $\sqrt[3]{40}$, $\sqrt[4]{5}$



② $\sqrt{11}$, $\sqrt[3]{23}$, $\sqrt{2}$, $\sqrt[3]{-3}$, $\sqrt[4]{30}$



[EX13] Determine whether each number is rational or irrational.

① $\sqrt{9}$ ② $\sqrt[3]{-27}$ ③ $\sqrt{6400}$ ④ $\sqrt{125}$

⑤ $\sqrt[3]{\dfrac{64}{125}}$ ⑥ $\sqrt[3]{0.27}$ ⑦ $\sqrt{\dfrac{25}{169}}$ ⑧ $\sqrt{432}$

[EX14] Given $\sqrt{2} \approx 1.41$ and $\sqrt{5} \approx 2.23$, estimate the value of the following without using a calculator.

① $\sqrt{200}$ ② $\sqrt{0.02}$ ③ $\sqrt{20}$

◆ Removing Brackets

$+(A-B-C)=A-B-C$ $\qquad$ $-(A-B-C)=-A+B+C$

(Example 4) Simplify.

① $(7x^2-8x+12)+(3x^2-4x+5)$

(Method 1) Horizontal Addition

$=7x^2\boxed{-8x}+12+3x^2\boxed{-4x}+5$

$=10x^2-12x+17$

(Method 2) Vertical Addition

$$\begin{array}{rrrr} & 7x^2 & -8x & +12 \\ + & 3x^2 & -4x & +5 \\ \hline = & 10x^2 & -12x & +17 \end{array}$$

② $(5a^2+4b-13)-(2a^2-3b-6)$

(Method 1) Horizontal Subtraction

$=\boxed{5a^2}+4b-13\boxed{-2a^2}+3b+6$

$=3a^2+7b-7$

(Method 2) Vertical Subtraction

$$\begin{array}{rrrr} & 5a^2 & +4b & -13 \\ - & 2a^2 & -3b & -6 \\ \hline = & 3a^2 & +7b & -7 \end{array}$$

[EX8] Simplify.

① $(6x-2y)+(-7x+11y)$

② $(16ab-5a)-(8a+13ab)$

③ $(2m-4n)-(5n-m)$

④ $-(11x+15)+(15x+9)$

⑤ $(3x+4y-5)+(-5x+7y-8)$

⑥ $(5x^2-8x+2)-(-12x^2+6x-3)$

⑦ $(-4a^2+3a+2)-(a^2-2a-3)$

⑧ $(2x^2-3x+7)+(-3x^2+6x-4)$

⑲ $(xy+6)(xy-3)$ ⑳ $(x^2+4)(4x^2+9)$ ㉑ $(t^3-8)(t^3+2)$

㉒ $(8p^3-1)(p^3+8)$ ㉓ $(3a^2-5b^2)(5a^2-2b^2)$ ㉔ $(4x^2+5y)(7x^2+6y)$

㉕ $(2m^2-3n^2)(4m^2-3n^2)$ ㉖ $\left(2x+\frac{y}{3}\right)\left(3x-\frac{y}{2}\right)$ ㉗ $\left(4a-\frac{b}{5}\right)\left(5a+\frac{b}{4}\right)$

(Example 6) Expand and simplify:

① $3x(3x-4)(x+5)$

$= 3x(3x^2+11x-20)$

$= 9x^3+33x^2-60x$

Multiply the two binomials first.
Then multiply the result by $3x$.

② $(2x+3)^3$

$= (2x+3)(2x+3)^2 = (2x+3)(4x^2+12x+9) = 8x^3+24x^2+18x+12x^2+36x+27$

$= 8x^3+36x^2+54x+27$

[EX8] Expand and simplify.

① $2x(x-3)(2x+5)$ ② $3(2x+7)(5x-4)$

③ $(x-y)(x+y)^2$

(Example 2) Factor.

① $2x^2 + 5x - 3$

$$\begin{array}{ccc} 2 & 3 & \to 3 \\ 1 & -1 & \to -2 \\ \hline & & 1 \end{array} \times \qquad \begin{array}{ccc} 2 & -3 & \to -3 \\ 1 & 1 & \to 2 \\ \hline & & -1 \end{array} \times \qquad \begin{array}{ccc} 2 & 1 & \to 1 \\ 1 & -3 & \to -6 \\ \hline & & -5 \end{array} \times \qquad \begin{array}{ccc} 2 & -1 & \to -1 \\ 1 & 3 & \to 6 \\ \hline & & 5 \end{array} \checkmark$$

$\therefore\ 2x^2 + 5x - 3 = \underline{(2x-1)(x+3)}$

③ $2x^2 - x - 3$

$$\begin{array}{ccc} 2 & -3 & \to -3 \\ 1 & 1 & \to 2 \\ \hline & & -1 \end{array} \checkmark$$

$= \underline{(2x-3)(x+1)}$

④ $6x^2 - 5xy - 6y^2$

$$\begin{array}{ccc} 2 & -3 & \to -9 \\ 3 & 2 & \to 4 \\ \hline & & -5 \end{array} \checkmark$$

$= \underline{(2x-3y)(3x+2y)}$

[EX2] Factor.

① $2x^2 - 3x - 2$

② $2x^2 + 3x - 5$

③ $2a^2 - a - 6$

④ $3x^2 + 5x - 2$

⑤ $3x^2 - x - 2$

⑥ $3m^2 - 2m - 5$

⑦ $-4x^2 - x + 3$

⑧ $4y^2 + 4y - 3$

⑨ $-5t^2 + 3t + 8$

3. Relations and Functions

★ Relations

◆ **Relation**

A relation is a set of ordered pairs $(input,\ output)$ or $(x,\ y)$.

◆ **Domain**

The set of the input values or x values in the ordered pairs is called the **domain** of the relation. x is called the **independent variable**.

◆ **Range**

The set of the output values or y values in the ordered pairs is called the **range** of the relation. y is called the **dependent variable** because its value depends on the value of x.

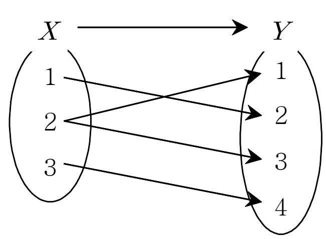
(Example 1) Consider the relation $\{(1,\ 2),\ (2,\ 1),\ (3,\ 4),\ (2,\ 3)\}$.

① Express the relation:

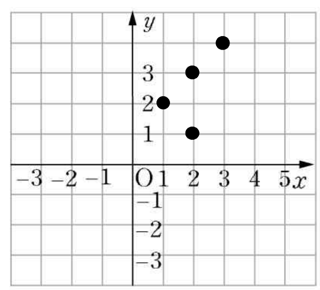
(1) as a table

x	y
1	2
2	1
3	4
2	3

(2) as an arrow diagram



(3) as a graph



② Find the domain and range.

Domain $=\{1,\ 2,\ 3\}$, Range $=\{1,\ 2,\ 3,\ 4\}$

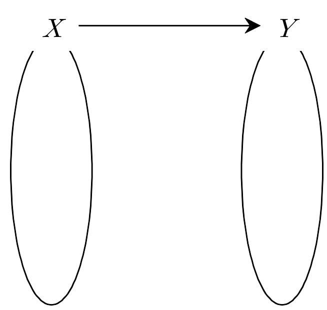
[EX1] Consider the relation $\{(-2,\ -2),\ (0,\ 1),\ (3,\ 1),\ (5,\ 3)\}$.

① Express the relation:

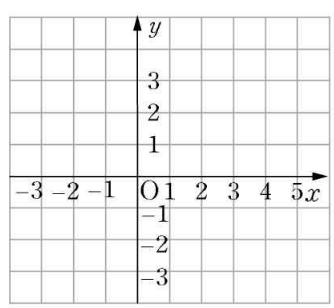
(1) as a table

x	y

(2) as an arrow diagram

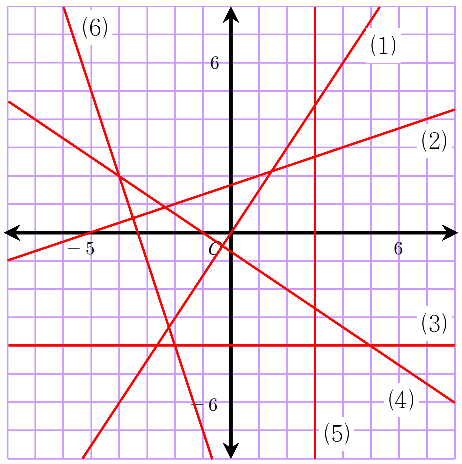


(3) as a graph



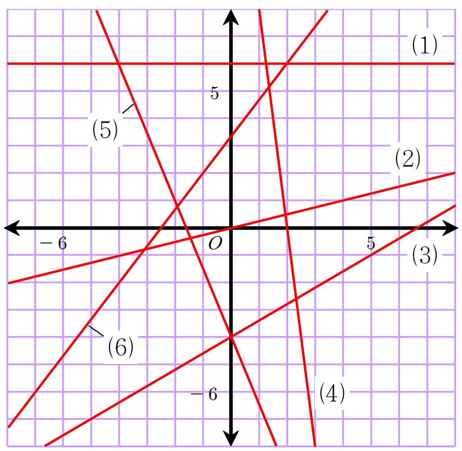
② Find the domain and range.

[EX4] Find the slope of each line.

①


(1) ______ (2) ______ (3) ______

(4) ______ (5) ______ (6) ______

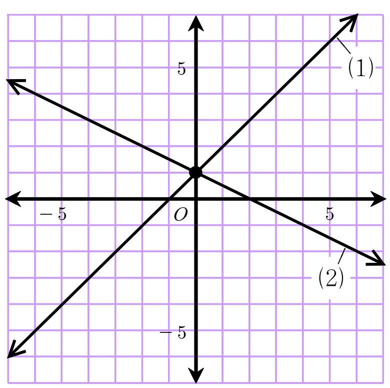
②


(1) ______ (2) ______ (3) ______

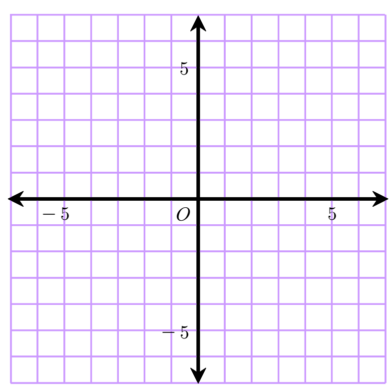
(4) ______ (5) ______ (6) ______

[EX5] Graph the line passing through each point with each given slope.

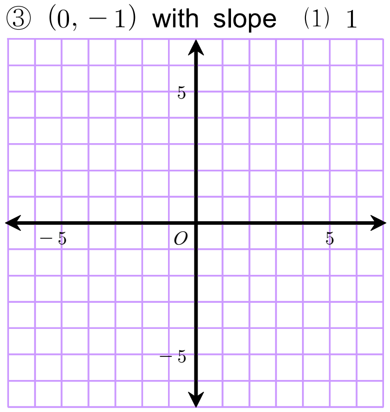
① $(0, 1)$ with slope (1) 1 (2) $-\frac{1}{2}$



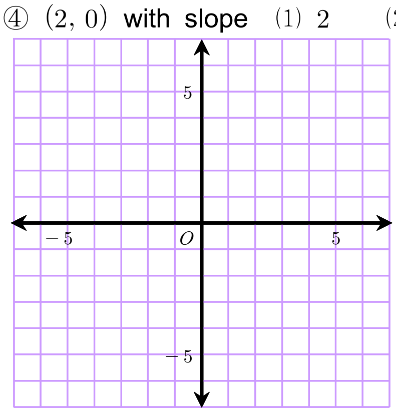
② $(0, 0)$ with slope (1) 2 (2) -1



③ $(0, -1)$ with slope (1) 1 (2) 3

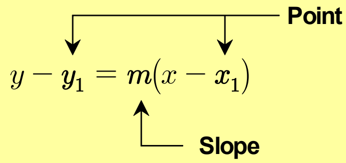


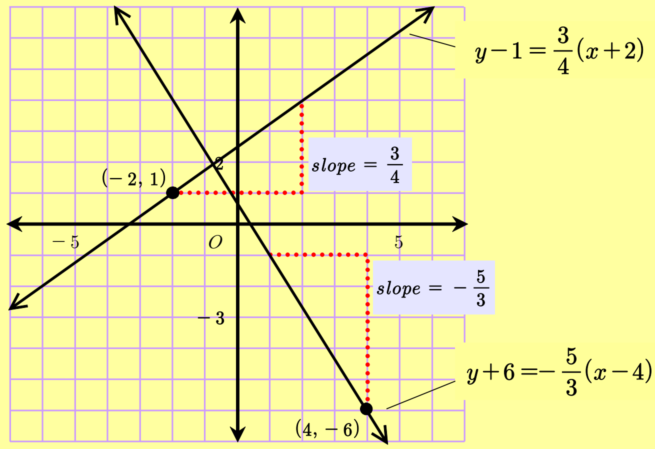
④ $(2, 0)$ with slope (1) 2 (2) -2



★ Point-Slope Form of Linear Equations, $y - y_1 = m(x - x_1)$

The equation of a line through (x_1, y_1) with a slope of m is $y - y_1 = m(x - x_1)$.





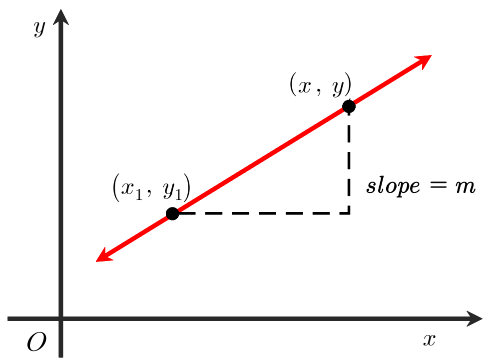
◆ Slope

- If $m > 0$, the line rises from left to right.
- If $m < 0$, the line falls from left to right.

[Proof]

Consider a line through (x_1, y_1) with a slope of m.

Let (x, y) be an arbitrary point on the line.



Slope of the line: $m = \dfrac{y - y_1}{x - x_1}$

Multiply both sides by $x - x_1$: $m(x - x_1) = y - y_1$

$\therefore\ y - y_1 = m(x - x_1)$

(Example 1) Consider the equation $y - 2 = -\dfrac{2}{3}(x+1)$.

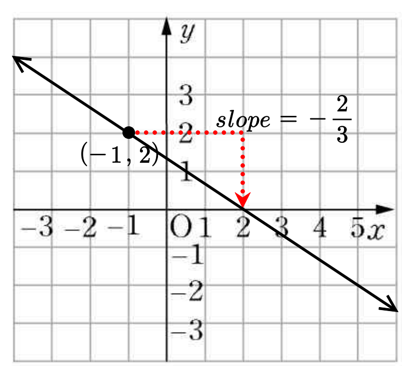
(1) Find the slope of the line and identify a point on the line. (2) Graph the line.

(1) $y-2 = -\dfrac{2}{3}(x+1) \quad \Leftarrow \quad y - y_1 = m(x - x_1)$

$m = -\dfrac{2}{3}$, $(x_1, y_1) = (-1, 2)$

$\therefore$ Slope $= -\dfrac{2}{3}$, Point: $(-1, 2)$

(2)



(Example 2) Maya earns a monthly base salary plus a commission on each beauty product she sells. In one month, she sold \$14 500 worth of products and earned \$3034. The following month, her sales totaled \$21 750, and her earnings were \$4049.

① Find her commission rate.

⇒ The line passes through the points $(14\,500, 3034)$ and $(21\,750, 4049)$.

$$\frac{4049-3034}{21\,750-14\,500}=\frac{1015}{7250}=0.14$$

∴ Maya's commission rate is 14%.

② Write the equation representing Maya's monthly earnings E, in dollars, in terms of her product sales x, in dollars.

⇒ Write the equation in point-slope form:

$$E-3034=0.14(x-14\,500)$$
$$E-3034=0.14x-2030$$
$$E=0.14x-2030+3034$$
$$\therefore\ E=0.14x+1004$$

③ Find her monthly base salary.

⇒ When $x=0$, $E=0.14(0)+1004=1004$

∴ Maya's monthly base salary is $\$1004$.

④ If her total sales this month is \$35 000, how much will she earn?

⇒ When $x=45\,000$, $E=0.14(45\,000)+1004=7034$

∴ Maya will earn $\$7034$.

[EX14] A sneaker store employee earns a monthly base salary plus a commission on every pair of sneakers he sells. In March, he sold \$28 800 worth of sneakers, earning him \$4 448. The following month, he sold \$36 000 worth of sneakers and earned \$5 168.

① Find his commission rate.

② Write the equation representing the employee's earnings E, in dollars, in terms of his sales x, in dollars.

③ Find his monthly base salary.

④ If his total sales this month is \$30 000, how much will he earn?

★ Solving Systems of Linear Equations by Elimination

◆ Solving Systems of Equations by Elimination

Step 1. Rewrite both equations in standard form, $Ax + By = C$.

Step 2. Multiply one or more of the equations by a non-zero number to make the coefficients of one variable equal.

Step 3. Add or subtract the equations to eliminate one variable, then solve for the other variable.

Step 4. Substitute the solution into either original equation, then solve for the eliminated variable.

Step 5. Write the solution as an ordered pair.

(Example 1) Solve by elimination: $\begin{cases} 2x + y = 9 \\ x - 2y = 2 \end{cases}$

(Method 1) **Addition**

$$2x + y = 9 \xrightarrow[\text{(Multiply by } 2)]{} 4x + \cancel{2y} = 18$$

$$x - 2y = 2 \xrightarrow[\text{(Leave alone)}]{} x - \cancel{2y} = 2$$

$$+ \qquad 5x = 20, \quad x = 4$$

Add the equations to eliminate the variable y, then solve for x.

Substitute 4 for x in one of the original equations: $2(4) + y = 9$

$y = 1$ $\qquad \therefore$ The solution is $(4, 1)$.

(Method 2) **Subtraction**

$$2x + y = 9 \xrightarrow[\text{(Leave alone)}]{} \cancel{2x} + y = 9$$

$$x - 2y = 2 \xrightarrow[\text{(Multiply by } 2)]{} \cancel{2x} - 4y = 4$$

$$- \qquad 5y = 5, \quad y = 1$$

Subtract the equations to eliminate the variable x, then solve for y.

Substitute 1 for y in one of the original equations: $x - 2(1) = 2$

$x = 4$ $\qquad \therefore$ The solution is $(4, 1)$.

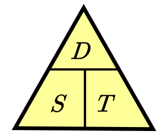
[EX1] Solve each system of equations by elimination.

① $\begin{cases} x + 2y = 11 \\ x - y = -4 \end{cases}$

② $\begin{cases} 4x - 3y = 11 \\ 2x + 3y = 1 \end{cases}$

(Example 5) A motorboat travels 48 *km* upstream in 3 hours and returns downstream in 2 hours. Find the speed of the motorboat in still water and the speed of the current.

Let x be the speed of the motorboat, and y be the speed of the current.



	Distance (km)	**Speed** (kph)	**Time** (h)
Upstream	48	$x-y$	3
Downstream	48	$x+y$	2

$\boxed{D = S \cdot T}$

$3(x-y) = 48 \Rightarrow x-y=16$ ①

$2(x+y) = 48 \Rightarrow x+y=24$ ②

①＋②

$$\begin{array}{r} x - \cancel{y} = 16 \\ + \quad x + \cancel{y} = 24 \\ \hline 2x \quad = 40 \end{array} \qquad \therefore\ x = 20$$

① $20 - y = 16 \qquad \therefore\ y = 4$

$\therefore$ The speed of the boat in still water is $20\ km/h$ and the speed of the current is $4\ km/h$.

[EX5] Solve each of the following problems. Show your work.

① A plane travels 3360 *km* from Vancouver to Toronto in 5 hours with a tail wind. The return trip takes 6 hours with a head wind. If the wind speed is constant, find the speed of the plane in still air and the speed of the wind.

② Two motorboats are 280 *km* apart and are traveling toward each other. One boat's speed is 40 *km/h* faster than the other. If they meet after 2 hours, find the speed of each motorboat.

③ Philip traveled 970 *km* from Vancouver to Calgary, by car and train. He traveled part of the journey by car at 80 *km/h*, then took a train for the remaining distance at 100 *km/h*. If the entire trip took 10.5 hours, how many hours did he spend driving, and how many did he spend on the train?

★ Arithmetic Series

◆ A **series** is the sum of all terms in a sequence.

(e.g.) $1,\ 3,\ 5,\ 7,\ 9,\ \cdots$ is a sequence.

$1+3+5+7+9+\cdots$ is a series.

The sum of the first n terms of a series is denoted by S_n.

$S_1 = t_1$

$S_2 = t_1 + t_2$

$S_3 = t_1 + t_2 + t_3$

$S_4 = t_1 + t_2 + t_3 + t_4$

$\vdots$

$S_n = t_1 + t_2 + t_3 + t_4 + \cdots + t_n$

◆ An **arithmetic series** is the sum of the terms of an arithmetic sequence**.** The sum of the first n terms of an arithmetic series is given by

$$S_n = \frac{n(a+t_n)}{2} = \frac{n[2a+(n-1)d]}{2}$$

where S_n is the sum of the first n terms, t_n is the nth term, a is the first term, n is the number of terms, and d is the common difference.

[Proof]

Let S_{100} be the sum of the first 100 natural number.

$$S_{100} = 1 + 2 + 3 + 4 + \cdots + 97 + 98 + 99 + 100$$

101

101

101

101

There are 50 pairs of 101.

$$\therefore\ S_{100} = \underbrace{\boxed{50}}_{\frac{Nmber\ of\ terms}{2}} \times \underbrace{\boxed{101}}_{(First\ term\ +\ Last\ term)} = 5050$$

The same method can be used to find S_n.

Let S_n be the sum of the first n terms of the arithmetic series.

$S_n = a + (a+d) + (a+2d) + \cdots + (t_n - 2d) + (t_n - d) + t_n$ where t_n is the last term.

$a+t_n$

$a+t_n$

$a+t_n$

There are $\frac{n}{2}$ pairs of $(a+t_n)$. $\therefore\ S_n = \frac{n(a+t_n)}{2}$ (1)

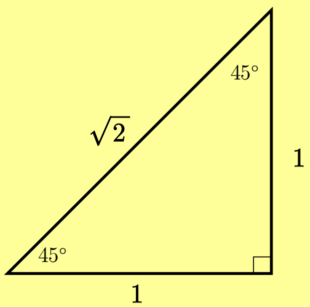
Substitute $t_n = a+(n-1)d$ for t_n in (1).

$$S_n = \frac{n(a+t_n)}{2} = \frac{n[a+a+(n-1)d]}{2} = \frac{n[2a+(n-1)d]}{2} \qquad \therefore\ S_n = \frac{n(2a+(n-1)d)}{2} \quad (2)$$

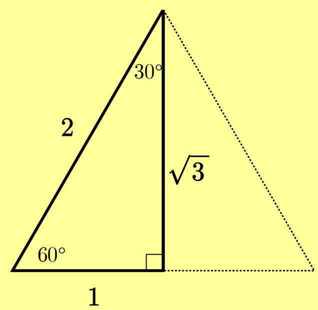
8. Right Triangle Trigonometry

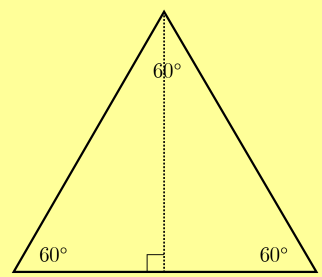
★ Special Right Triangles

◆ **45°-45°-90° (Isosceles Right Triangle)**



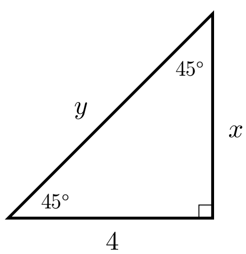
◆ **30°-60°-90° (Half an Equilateral Triangle)**





(Example 1) Find the values of x and y, using the ratio of the sides.

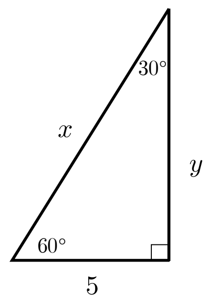
①



$\frac{x}{4} = \frac{1}{1}$ $\quad \therefore\ x = 4$

$\frac{y}{4} = \frac{\sqrt{2}}{1}$ $\quad \therefore\ y = 4\sqrt{2}$

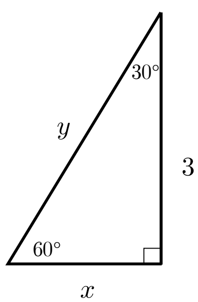
②



$\frac{x}{5} = \frac{2}{1}$ $\quad \therefore\ x = 10$

$\frac{y}{5} = \frac{\sqrt{3}}{1}$ $\quad \therefore\ y = 5\sqrt{3}$

③



$\frac{x}{3} = \frac{1}{\sqrt{3}}$

$x = \frac{3}{\sqrt{3}} = \frac{3 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$

(Rationalize denominator)

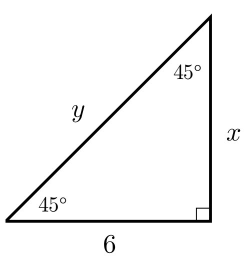
$\therefore\ x = \sqrt{3}$

$\frac{y}{x} = \frac{2}{1} \quad \Rightarrow \quad y = 2x$

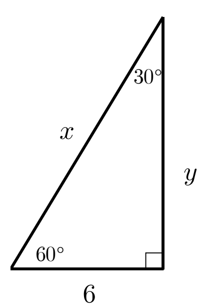
$\therefore\ y = 2\sqrt{3}$

[EX1] Find the values of x and y, using the ratio of the sides. Do not use a calculator.

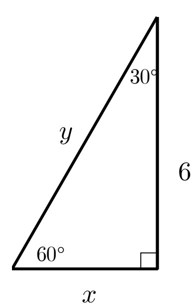
①



②



③

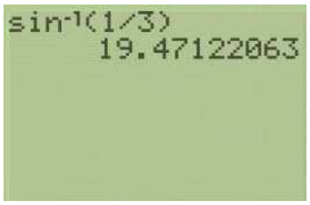


(Example 4) Use a calculator to solve for θ, to one decimal place. $(0° \le \theta \le 90°)$.

$$\sin\theta = \frac{1}{3}$$

Step 1. Press **MODE**. Scroll down to **Radian Degree**, select **Degree**, then press **ENTER**.

Step 2. Press **2nd** **MODE**. Press **2nd** **SIN** and type $1/3$, then press **ENTER**.



$\therefore \ \theta = 19.5°$

[EX5] Use a calculator to solve for θ, in degrees to one decimal place. $(0° \le \theta \le 90°)$.

① $\sin\theta = \frac{3}{4}$

$\theta = \sin^{-1}\left(\qquad \right)$

$\therefore \ \theta =$

② $\cos\theta = 0.4728$

$\theta = \cos^{-1}\left(\qquad \right)$

$\therefore \ \theta =$

③ $\tan\theta = \frac{4}{7}$

$\theta = \tan^{-1}\left(\qquad \right)$

$\therefore \ \theta =$

④ $\tan\theta = 1.4035$

⑤ $\cos\theta = \frac{2}{3}$

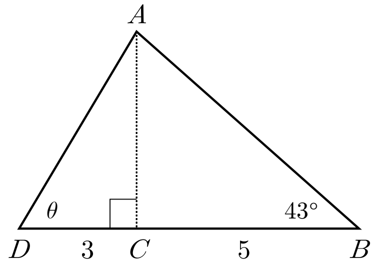
⑥ $\sin\theta = 0.5$

⑦ $\sin\theta = 0.4537$

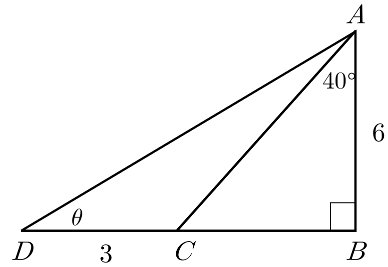
⑧ $\tan\theta = 4.2644$

⑨ $\cos\theta = 1$

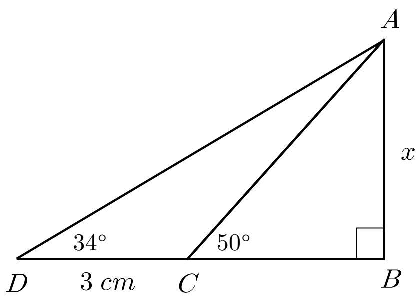
⑤



⑥



(Example 9) Use a calculator to find the length of side x in the diagram, to one decimal place.



In $\triangle ABC$, $\angle CAB = 40°$.

In $\triangle ABD$, $\angle DAB = 56°$.

In $\triangle ABC$, $\tan 40° = \dfrac{CB}{x} \Rightarrow CB = x\tan 40°$

In $\triangle ABD$, $\tan 56° = \dfrac{DB}{x} \Rightarrow DB = x\tan 56°$

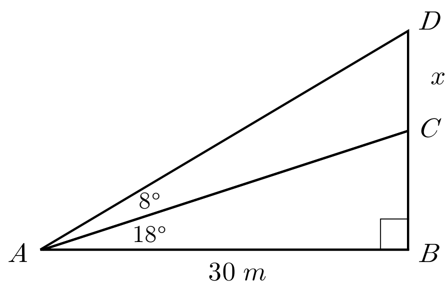
$DB - CB = 3 \Rightarrow x\tan 56° - x\tan 40° = 3$

Factor out the x. $x(\tan 56° - \tan 40°) = 3$

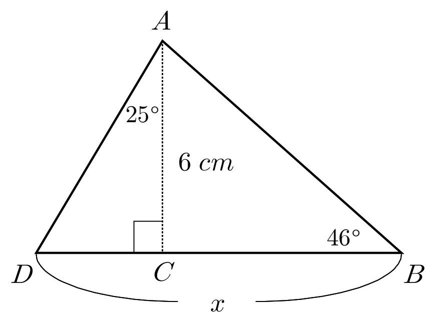
$$\therefore\ x = \frac{3}{\tan 56° - \tan 40°} = 4.66... \approx \underline{4.7\ cm}$$

[EX10] Use a calculator to find the length of side x in each diagram, to one decimal place.

①



②



★ Applications of Trigonometry

◆ Solving Applications of Trigonometry

Step 1. Read the problem carefully to identify the given informations (angles and sides) and the value to be found.

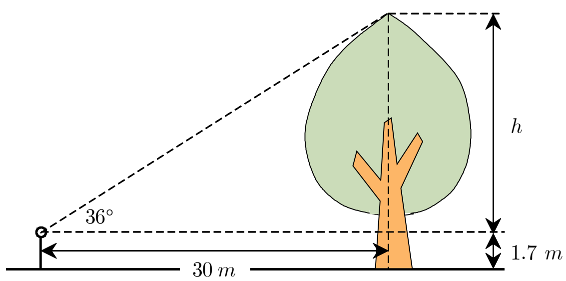
Step 2. Draw a clear diagram with the given informations and the unknown value. If necessary, draw auxiliary lines to form a right triangle.

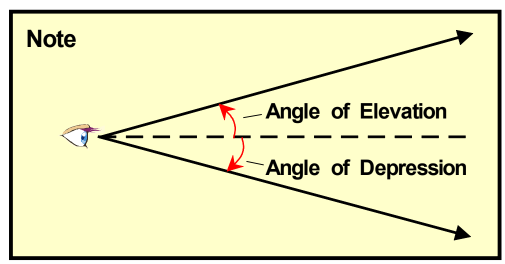
Step 3. Choose one of the trigonometric ratios: $\sin\theta$, $\cos\theta$, or $\tan\theta$.

Step 4. Setup and solve the equation.

Step 5. Check the solutions by using the original words of the problem.

(Example 1) A student stands 30 *m* from the base of an oak tree. His eye level is 1.7 *m* above the ground and he measures the angle of elevation to the top of the tree to be 36°. Find the height of the oak tree, to the nearest tenth of a meter.





$$\tan\theta = \frac{Opposite}{Adjacent} \Rightarrow \tan 36° = \frac{h}{30} \Rightarrow h = 30\tan 36° = 21.79...$$

Height of the oak tree $=$ Eye level $+\ h$

$= 1.7 + 21.79...$

$= 23.49...$ ∴ The height of the oak tree is $23.5\,m$.

[EX1] ─ [EX24] Round your answer to the nearest tenth.

[EX1] A camera is placed on the ground 80 *m* from the base of a flagpole. If the angle of elevation to the top of the flagpole is 30°, find the height of the pole.

[EX2] From a fishing boat, the angle of elevation to the top of a cliff is 12°. If the distance from the boat to the base of the cliff is 320 *m*, find the height of the cliff.

SOLUTIONS

[EX9]

① $$\begin{array}{c|cc} 2 & 78 & 102 \\ \hline 3 & 39 & 51 \\ \hline & 13 & 17 \end{array} \quad \therefore\ GCF = 2\cdot 3 = 6$$

② $$\begin{array}{c|cc} 2 & 66 & 154 \\ \hline 11 & 33 & 77 \\ \hline & 3 & 7 \end{array} \quad \therefore\ GCF = 2\cdot 11 = 22$$

③ $$\begin{array}{c|cc} 2 & 198 & 234 \\ \hline 9 & 99 & 117 \\ \hline & 11 & 13 \end{array} \quad \therefore\ GCF = 2\cdot 9 = 18$$

④ $$\begin{array}{c|ccc} 2 & 56 & 98 & 70 \\ \hline 7 & 28 & 49 & 35 \\ \hline & 4 & 7 & 5 \end{array} \quad \therefore\ GCF = 2\cdot 7 = 14$$

⑤ $$\begin{array}{c|ccc} 4 & 396 & 540 & 504 \\ \hline 9 & 99 & 135 & 126 \\ \hline & 11 & 15 & 14 \end{array} \quad \therefore\ GCF = 4\cdot 9 = 36$$

[EX10]

① $18 = 2\cdot 3^2$
$24 = 2^3\cdot 3$
$\therefore\ LCM = 2^3\cdot 3^2 = 72$

② $60 = 2^2\cdot 3\cdot 5$
$48 = 2^4\cdot 3$
$\therefore\ LCM = 2^4\cdot 3\cdot 5 = 240$

③ $75 = 3\cdot 5^2$
$45 = 3^2\cdot 5$
$\therefore\ LCM = 3^2\cdot 5^2 = 225$

④ $12 = 2^2\cdot 3$
$15 = 1\cdot 3\cdot 5$
$18 = 2\cdot 3^2$ $\therefore\ LCM = 2^2\cdot 3^2\cdot 5 = 180$

[EX11]

① $$\begin{array}{c|cc} 6 & 54 & 60 \\ \hline & 9 & 10 \end{array} \quad \therefore\ LCM = 6\cdot 9\cdot 10 = 540$$

② $$\begin{array}{c|cc} 8 & 64 & 80 \\ \hline 2 & 8 & 10 \\ \hline & 4 & 5 \end{array} \quad \therefore\ LCM = 8\cdot 2\cdot 4\cdot 5 = 320$$

③ $$\begin{array}{c|cc} 8 & 72 & 96 \\ \hline 3 & 9 & 12 \\ \hline & 3 & 4 \end{array} \quad \therefore\ LCM = 8\cdot 3\cdot 3\cdot 4 = 288$$

④ $$\begin{array}{c|ccc} 3 & 24 & 45 & 36 \\ \hline 4 & 8 & 15 & 12 \\ \hline 3 & 2 & 15 & 3 \\ \hline & 2 & 5 & 1 \end{array} \quad \therefore\ LCM = 3\cdot 4\cdot 3\cdot 2\cdot 5 = 360$$

⑤ $$\begin{array}{c|ccc} 6 & 48 & 72 & 90 \\ \hline 4 & 8 & 12 & 15 \\ \hline 3 & 2 & 3 & 15 \\ \hline & 2 & 1 & 5 \end{array} \quad \therefore\ LCM = 6\cdot 4\cdot 3\cdot 2\cdot 5 = 720$$

[EX12]

① $=\dfrac{3}{4}$ ② $=\dfrac{5}{6}$ ③ $=\dfrac{7}{3}$ ④ $=\dfrac{11}{19}$ ⑤ $=\dfrac{3}{4}$ ⑥ $=\dfrac{17}{19}$

[EX13]
$$\begin{array}{c|ccc} 6 & 48 & 36 & 30 \\ \hline & 8 & 6 & 5 \end{array} \quad \therefore\ GCF = 6\ baskets$$

[EX14]
$$\begin{array}{c|ccc} 4 & 288 & 144 & 160 \\ \hline 4 & 72 & 36 & 40 \\ \hline & 18 & 9 & 10 \end{array} \quad \therefore\ GCF = 4\cdot 4 = 16$$

[EX15]
$$\begin{array}{c|cc} 8 & 456 & 768 \\ \hline 3 & 57 & 96 \\ \hline & 19 & 32 \end{array} \quad \therefore\ GCF = 24 \ \Rightarrow\ 24\ cm\ by\ 24\ cm \ \Rightarrow\ \frac{456\times 768}{24\times 24} = 608\ tiles$$

[EX16]
$$\begin{array}{c|ccc} 3 & 12 & 15 & 18 \\ \hline 2 & 4 & 5 & 6 \\ \hline & 2 & 5 & 3 \end{array}$$
$LCM = 3\cdot 2\cdot 2\cdot 5\cdot 3$
$= 180\ min$
$\therefore\ 10:00\ AM$

[EX17]
$$\begin{array}{c|ccc} 3 & 6 & 4 & 9 \\ \hline 2 & 2 & 4 & 3 \\ \hline & 1 & 2 & 3 \end{array}$$
$LCM = 3\cdot 2\cdot 3\cdot 2$
$= 36\ days$
$1+36-31(Jan) = 6$
$\therefore\ Feb\ 6th$

[EX18]
$$\begin{array}{c|cccc} 3 & 6 & 12 & 21 & 35 \\ \hline 2 & 2 & 4 & 7 & 35 \\ \hline 7 & 1 & 2 & 7 & 35 \\ \hline & 1 & 2 & 1 & 5 \end{array}$$
$LCM = 3\cdot 2\cdot 7\cdot 2\cdot 5$
$= 420 \quad \Rightarrow \quad 420,\ 840,\ 1260,\ 1680,\ .....$

$\therefore$ The smallest 4-digit number is 1260.

★ Square Roots and Cube Roots: p22

[EX1] ① Yes ② Yes ③ Yes ④ No $(2592 = 4^2\cdot 2\cdot 9^2)$ ⑤ Yes

[EX2] ① $\sqrt{121} = 11$ ② $\sqrt{729} = 27$ ③ $\sqrt{1024} = 32$ ④ $\sqrt{625} = 25$ ⑤ $\sqrt{6400} = 80$ ⑥ $\sqrt{2304} = 48$

⑦ $\sqrt{12100} = 110$ ⑧ $\sqrt{1.44} = 1.2$ ⑨ $\sqrt{0.0169} = 0.13$ ⑩ $\sqrt{\dfrac{9}{16}} = \dfrac{3}{4}$ ⑪ $\sqrt{\dfrac{25}{81}} = \dfrac{5}{9}$ ⑫ $\sqrt{\dfrac{441}{100}} = \dfrac{21}{10}$

[EX2]

① $2x^2 - 3x - 2$
$$\begin{array}{ccc} 1 & -2 & \to -4 \\ 2 & 1 & \to 1 \\ \hline & & -3 \end{array}$$
$= (x-2)(2x+1)$

② $2x^2 + 3x - 5$
$$\begin{array}{ccc} 1 & -1 & \to -2 \\ 2 & 5 & \to 5 \\ \hline & & 3 \end{array}$$
$= (x-1)(2x+5)$

③ $2a^2 - a - 6$
$$\begin{array}{ccc} 2 & 3 & \to 3 \\ 1 & -2 & \to -4 \\ \hline & & -1 \end{array}$$
$= (2a+3)(a-2)$

④ $3x^2 + 5x - 2$
$$\begin{array}{ccc} 1 & 2 & \to 6 \\ 3 & -1 & \to -1 \\ \hline & & 5 \end{array}$$
$= (x+2)(3x-1)$

⑤ $3x^2 - x - 2$
$$\begin{array}{ccc} 1 & -1 & \to -3 \\ 3 & 2 & \to 2 \\ \hline & & -1 \end{array}$$
$= (x-1)(3x+2)$

⑥ $3m^2 - 2m - 5$
$$\begin{array}{ccc} 1 & 1 & \to 3 \\ 3 & -5 & \to -5 \\ \hline & & -2 \end{array}$$
$= (m+1)(3m-5)$

⑦ $=-(4x^2 + x - 3)$
$$\begin{array}{ccc} 1 & 1 & \to 4 \\ 4 & -3 & \to -3 \\ \hline & & 1 \end{array}$$
$= -(x+1)(4x-3)$

⑧ $4y^2 + 4y - 3$
$$\begin{array}{ccc} 2 & -1 & \to -2 \\ 2 & 3 & \to 6 \\ \hline & & 4 \end{array}$$
$= (2y-1)(2y+3)$

⑨ $=-(5t^2 - 3t - 8)$
$$\begin{array}{ccc} 1 & 1 & \to 5 \\ 5 & -8 & \to -8 \\ \hline & & -3 \end{array}$$
$= -(t+1)(5t-8)$

⑩ $=-(3a^2 - 10a - 8)$
$$\begin{array}{ccc} 3 & 2 & \to 2 \\ 1 & -4 & \to -12 \\ \hline & & -10 \end{array}$$
$= -(3a+2)(a-4)$

⑪ $=-(6x^2 + x - 1)$
$$\begin{array}{ccc} 2 & 1 & \to 3 \\ 3 & -1 & \to -2 \\ \hline & & 1 \end{array}$$
$= -(2x+1)(3x-1)$

⑫ $=-(2x^2 - 13x - 7)$
$$\begin{array}{ccc} 2 & 1 & \to 1 \\ 1 & -7 & \to -14 \\ \hline & & -13 \end{array}$$
$= -(2x+1)(x-7)$

⑬ $25x^2 + 40xy + 16y^2$
$$\begin{array}{ccc} 5 & 4 & \to 20 \\ 5 & 4 & \to 20 \\ \hline & & 40 \end{array}$$
$= (5x+4y)^2$

⑭ $6m^2 + 11mn - 10n^2$
$$\begin{array}{ccc} 2 & 5 & \to 15 \\ 3 & -2 & \to -4 \\ \hline & & 11 \end{array}$$
$= (2m+5n)(3m-2n)$

⑮ $9a^2 - 4ab - 5b^2$
$$\begin{array}{ccc} 9 & 5 & \to 5 \\ 1 & -1 & \to -9 \\ \hline & & -4 \end{array}$$
$= (9a+5b)(a-b)$

⑯ $= (3a-5b)(3a+b)$

⑰ $= (2p+3q)(3p-2q)$

⑱ $= -(9x^2+9xy-4y^2) = -(3x-y)(3x+4y)$

⑲ $= (2x-3y)(3x-2y)$

⑳ $= (3x-7y)^2$

㉑ $= (2m+n)(3m-4n)$

㉒ $= -(4s^2-13st+10t^2)$
$= -(4s-5t)(s-2t)$

㉓ $= -(8x^2-13xy-6y^2)$
$= -(8x+3y)(x-2y)$

㉔ $= -(7a^2+17ab-12b^2)$
$= -(a+3b)(7a-4b)$

㉕ $= (7p-4q)(p+2q)$

㉖ $= (7x+3y)(x-2y)$

㉗ $= (3m+2n)(3m-4n)$

[EX3]

① $= (x^2+4)(5x^2-6)$

② $= (ab-6)(3ab+8)$

③ $= (t^3-15)(2t^3-5)$

④ $= (2x+3y)(6x-5y)$

⑤ $= (4p-5q)(8p+3q)$

⑥ $= -(24a^2-41ab+12b^2) = -(3a-4b)(8a-3b)$

[EX4]

① $= 2(x+2)(7x-4)$

② $= 5(x-2y)(7x+3y)$

③ $= -3(3m+2n)(3m-4n)$

④ $= 2x(4x^2-20x+25)$
$= 2x(2x-5)^2$

⑤ $= 4(10m^2+13mn-3n^2)$
$= 4(2m+3n)(5m-n)$

⑥ $= -x^2(12x^2+13xy-14y^2)$
$= -x^2(3x-2y)(4x+7y)$

⑦ $= 5x(9x^2+12x+4)$
$= 5x(3x+2)^2$

⑧ $= -2a(10a^2-9ab-36b^2)$
$= -2a(2a+3b)(5a-12b)$

⑨ $= ab(21a^2+13ab-20b^2)$
$= ab(3a+4b)(7a-5b)$

[EX5]

① Let $A = 2x+3y$. $3A^2 - A - 2$
$$\begin{array}{ccc} 3 & 2 & \to 2 \\ 1 & -1 & \to -3 \\ \hline & & -1 \end{array}$$
$= [3A+2][A-1]$
$= [3(2x+3y)+2][(2x+3y)-1]$
$= (6x+9y+2)(2x+3y-1)$

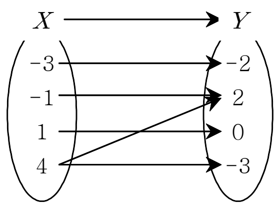
② Let $A = x^2+2x+2$. $A^2 - 6A + 5$
$$\begin{array}{ccc} 1 & -1 & \to -1 \\ 1 & -5 & \to -5 \\ \hline & & -6 \end{array}$$
$= [A-1][A-5]$
$= [(x^2+2x+2)-1][(x^2+2x+2)-5]$
$= (x^2+2x+1)(x^2+2x-3)$
$= (x+1)^2(x+3)(x-1)$

Chapter Review Exercises: p154

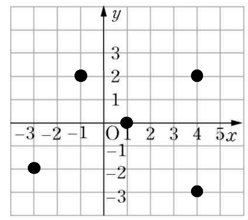
1. ① (1)

x	y
-3	-2
-1	2
1	0
4	2
4	-3

(2)



(3)

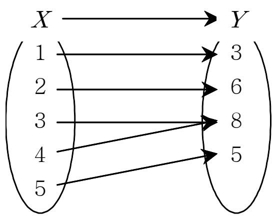


② $D: \{-3,\ -1,\ 1,\ 4\}$

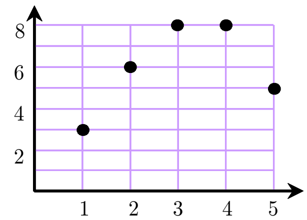
$R: \{-3,\ -2,\ 0,\ 2\}$

2. (1) $\{(1,\ 3),\ (2,\ 6),\ (3,\ 8),\ (4,\ 8),\ (5,\ 5)\}$

(2)



(3)



3. (1) $\{(-2,\ 3),\ (-1,\ 0),\ (0,\ 3),\ (2,\ -1),\ (3,\ 4),\ (5,\ -3)\}$

(2) $D: \{-2,\ -1,\ 0,\ 2,\ 3,\ 5\}$

$R: \{-3,\ -1,\ 0,\ 3,\ 4\}$

4. ① (1) $D: \{x \mid -3 \le x \le 5,\ x \in R\}$

$R: \{y \mid -1 \le y \le 3,\ y \in R\}$

(2) $D: [-3,\ 5],\quad R: [-1,\ 3]$

② (1) $D: \{x \mid -3 < x < 4,\ x \in R\}$

$R: \{y \mid -2 \le y < 3,\ y \in R\}$

(2) $D: (-3,\ 4),\quad R: [-2,\ 3)$

③ (1) $D: \{x \mid x \le 5,\ x \in R\}$

$R: \{y \mid -2 \le y < 0 \ \text{ or } \ y \ge 2,\ y \in R\}$

(2) $D: (-\infty,\ 5],\quad R: [-2,\ 0) \cup [2,\ \infty)$

5. ①

(1) $D: \{x \mid x \ge -1,\ x \in R\}$

$R: \{y \mid y \in R\}$

(2) Not a function.

②

(1) $D: \{x \mid x \in R\}$

$R: \{y \mid -2 < y < 3,\ y \in R\}$

(2) Function; 1-to-1.

③

(1) $D: \{x \mid -3 \le x < 3,\ x \in R\}$

$R: \{y \mid y = -2,\ 0,\ 3\}$

(2) Function; Not 1-to-1.

④

(1) $D: \{x \mid x < 0 \ \text{ or } \ x \ge 2,\ x \in R\}$

$R: \{y \mid y \in R\}$

(2) Function; 1-to-1.

6. ① $= 2[6(3) - 5]$

$= 2(18 - 5) = 26$

② $= -[6(x+2) - 5]$

$= -[6x + 12 - 5]$

$= -(6x + 7) = -6x - 7$

③ $= \dfrac{[6(x+h) - 5] - [6x - 5]}{h}$

$= \dfrac{6x + 6h - 5 - 6x + 5}{h} = \dfrac{6h}{h} = 6$

④ $6x - 5 = 19$

$6x = 24$

$x = 4$

7. ① $= (-2)^2 - 3(-2) + 4$

$= 4 + 6 + 4$

$= 14$

② $= (x-2)^2 - 3(x-2) + 4$

$= x^2 - 4x + 4 - 3x + 6 + 4$

$= x^2 - 7x + 14$

③ $= \dfrac{(x+h)^2 - 3(x+h) + 4 - [x^2 - 3x + 4]}{h}$

$= \dfrac{x^2 + 2xh + h^2 - 3x - 3h + 4 - x^2 + 3x - 4}{h}$

$= \dfrac{2xh + h^2 - 3h}{h} = 2x + h - 3$

④ $x^2 - 3x + 4 = 4$

$x^2 - 3x = 0$

$x(x-3) = 0$

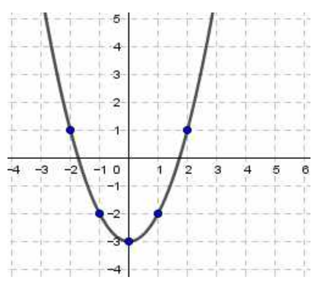
$x = 0,\ 3$

8. ① $= 4$ ② $= 0$ ③ $= -2$ ④ $x = -3,\ 3,\ 5$ ⑤ $x = -2,\ 2$ ⑥ $-1 \le x \le 1$ ⑦ $-2 < x < 2$

⑧ $= f(-2) - (-2) = 2 + 2 = 4$ ⑨ $= f(2) - 2 = 2 - 2 = 0$ ⑩ $= \dfrac{0 - 4}{4} = -1$

9. ① (1)

x	y
-2	1
-1	-2
0	-3
1	-2
2	1



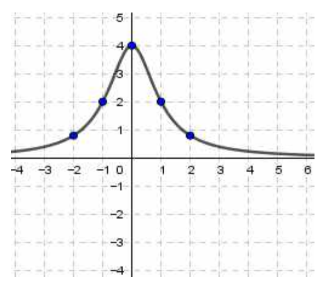
(2) Function

(3) $D: \{x \mid x \in R\}$

$R: \{y \mid y \ge -3,\ y \in R\}$

② (1)

x	y
-2	$4/5$
-1	2
0	4
1	2
2	$4/5$



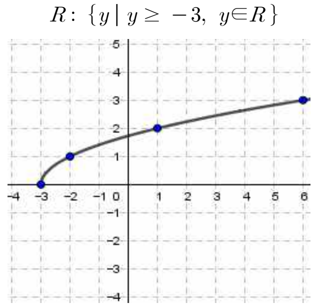
(2) Function

(3) $D: \{x \mid x \in R\}$

$R: \{y \mid 0 < y \le 4,\ y \in R\}$

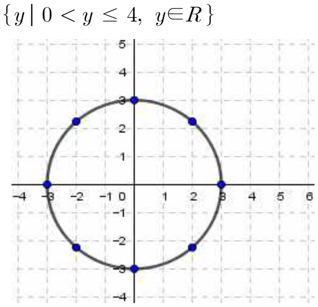
③ (1)

x	y
-3	0
-2	1
1	2
6	3



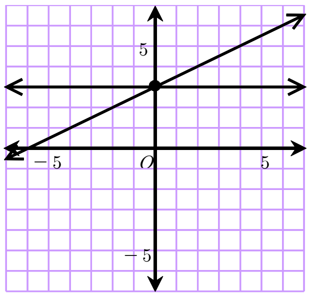
④ (1)

x	y
-3	0
-2	$\pm\sqrt{5} \approx \pm 2.24$
0	± 3
2	$\pm\sqrt{5} \approx \pm 2.24$
3	0

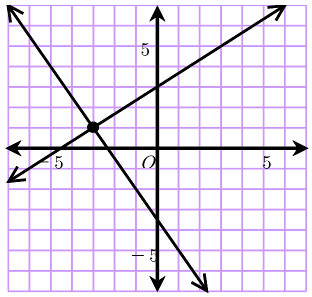


8. (1) $\frac{4}{3}$ (2) 0 (3) $\frac{1}{3}$ (4) $-\frac{1}{3}$ (5) -3 (6) undefined

9. ①



②



10. $\frac{a+5}{3-a} = 3$

$a+5 = 3(3-a)$

$a+5 = 9-3a$

$4a = 4 \quad \therefore\ a = 1$

11. Let the point be $(x,\ 0)$.

$\frac{0+4}{x-3} = \frac{1}{2}$

$x-3 = 8$

$x = 11 \quad \therefore\ (11,\ 0)$

12. Slope of AB = Slope of BC

$\frac{2-5}{1+3} = \frac{8-2}{k-1}$

$\frac{-3}{4} = \frac{6}{k-1} \quad \Rightarrow \quad -3(k-1) = 24$

$k-1 = -8$

$k = -7$

13. (1) $(0,\ 2000),\ (6,\ 920)$

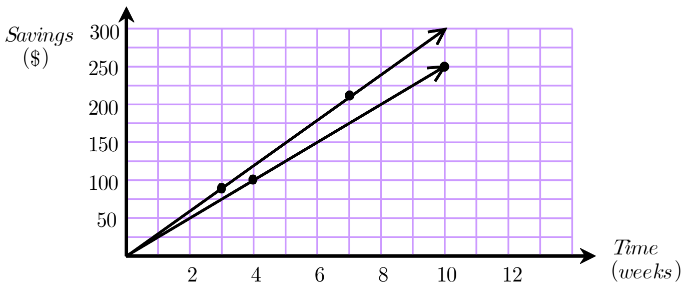
$\frac{\triangle y}{\triangle x} = \frac{920-2000}{0-6}$

$= \frac{-1080}{6} = -\$180/yr$

(2) Change in value $= -180 \cdot 3 = -540$

$920 - 540 = \$380$

14. ①



② Sasha: $m = \frac{210-90}{7-3} = \frac{120}{4} = \$30/week$

Benson: $m = \frac{250-100}{10-4} = \frac{150}{6} = \$25/week$

③ Sasha $= \$30/week \cdot 40 weeks = \$1200$

Benson$= \$25/week \cdot 40 weeks = \$1000$

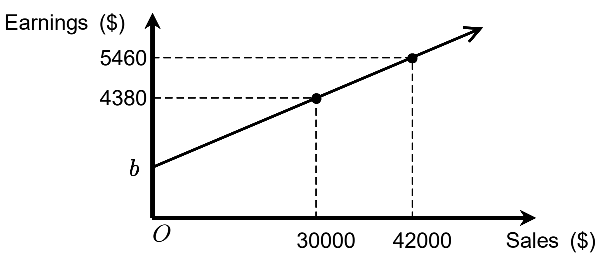
$\therefore$ Sasha saves \$200 more than Benson.

15. $32 + 4x \le 186$

$4x \le 154$

$x \le 38.5 \quad \therefore\ 38\ packs$

16. ①



② $\frac{5460-4380}{42\,000-30\,000} = \frac{1080}{12\,000} = 0.09 \quad \therefore\ 9\%$

③ $(0,\ b),\ (30\,000,\ 4380)$

$\frac{4380-b}{30\,000-0} = 0.09 \quad \rightarrow \quad 4380 - b = 2700$

$b = 4380 - 2700$

$= \$1680$

17. ① $\frac{5-2}{k+2} = \frac{-1+3}{7-4}$

$\frac{3}{k+2} = \frac{2}{3} \quad \Rightarrow \quad 2(k+2) = 9$

$2k+4 = 9$

$2k = 5$

$k = \frac{5}{2}$

② $\frac{5-2}{k+2} \cdot \frac{-1+3}{7-4} = -1$

$\frac{3}{k+2} \cdot \frac{2}{3} = -1 \quad \Rightarrow \quad \frac{2}{k+2} = -1$

$k+2 = -2$

$k = -4$

③ Let the point P be $(0,\ y)$.

$\frac{y-2}{0+2} \cdot \frac{2}{3} = -1$

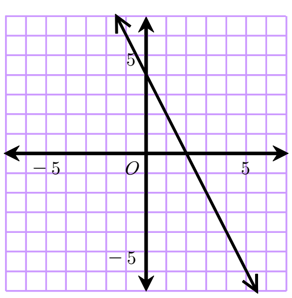
$\frac{y-2}{3} = -1 \quad \Rightarrow \quad y-2 = -3$

$y = -1$

$\therefore\ (0,\ -1)$

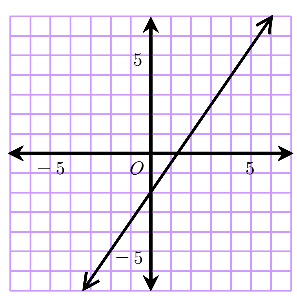
18. ①

x	y
0	4
1	2
2	0



②

x	y
0	−2
2	1
4	4



19. ① $3(k) - 4(-3) = 24$

$3k + 12 = 24$

$3k = 12$

$\therefore\ k = 4$

② $y + 3 = \frac{1}{2}(x-k), \quad (0,\ 3)$

$3 + 3 = \frac{1}{2}(0-k)$

$6 = -\frac{1}{2}k \quad \therefore\ k = -12$

③ $kx + 0 - 10 = 0 \qquad 0 + 4y + 12 = 0$

$x = \frac{10}{k} \qquad y = -3$

$\frac{10}{k} = -3 \quad \Rightarrow \quad \therefore\ k = -\frac{10}{3}$

[EX9] ① $-5y = -2x - 10$

$y = \frac{2}{5}x + 2 \quad \therefore\ m_2 = \frac{2}{5}$

$Let\ \ y = \frac{2}{5}x + b. \quad (5,\ -2)$

$-2 = \frac{2}{5}(5) + b$

$-2 = 2 + b, \quad b = -4$

$\therefore\ y = \frac{2}{5}x - 4$

② $9y = -3x + 15$

$y = -\frac{1}{3}x + \frac{5}{3} \quad \therefore\ m_2 = 3$

$Let\ \ y = 3x + b. \quad (-3,\ 4)$

$4 = -9 + b$

$b = 13$

$\therefore\ y = 3x + 13$

③ $-4y = -3x + 12$

$y = \frac{3}{4}x - 3 \quad \therefore\ m_2 = -\frac{4}{3}$

$3(0) - 4y = 12$

$y = -3$

$\therefore\ y\text{-}int = -3$

$\therefore\ y = -\frac{4}{3}x - 3$

④ $m_1 = \frac{5-8}{2+4} = \frac{-3}{6} = -\frac{1}{2}$

$\therefore\ m_2 = -\frac{1}{2}$

$Let\ \ y = -\frac{1}{2}x + b. \quad (5,\ 0)$

$0 = -\frac{5}{2} + b, \quad b = \frac{5}{2}$

$\therefore\ y = -\frac{1}{2}x + \frac{5}{2}$

[EX10] ① $m_1 = 4, \quad m_2 = -\frac{1}{4}$ $\quad\therefore$ perpendicular

② $m_1 = \frac{9}{6} = \frac{3}{2}, \quad m_2 = \frac{3}{2}$ $\quad\therefore$ parallel

③ $m_1 = \frac{4}{5}, \quad m_2 = \frac{10}{8} = \frac{5}{4}$ $\quad\therefore$ neither

④ $m_1 = -\frac{7}{3}, \quad m_2 = \frac{3}{7}$ $\quad\therefore$ perpendicular

[EX11] ① $\frac{k-6}{2-k} = \frac{1}{3}$

$3k - 18 = 2 - k$

$4k = 20, \quad \therefore\ k = 5$

② $6y = -4x + 12$

$y = -\frac{2}{3}x + 2$

$\therefore\ m_2 = \frac{3}{2}$

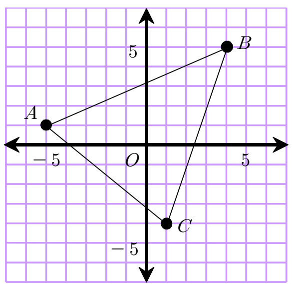
$\Rightarrow$ $\frac{k+1}{14-k} = \frac{3}{2}$

$2k + 2 = 42 - 3k$

$5k = 40, \quad \therefore\ k = 8$

[EX12]

(1)



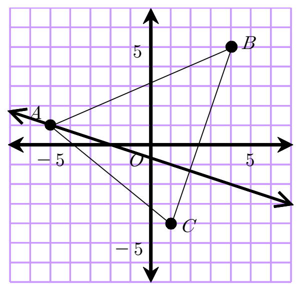
(2) $m_1 = \frac{-4-5}{1-4} = \frac{-9}{-3} = 3$

$Let\ y = 3x + b \quad (4,\ 5)$

$5 = 12 + b, \quad b = -7$

$\therefore\ y = 3x - 7$

(3)



(4) $m_2 = -\frac{1}{3}$

$Let\ y = -\frac{1}{3}x + b \quad (-5,\ 1)$

$1 = \frac{5}{3} + b, \quad b = -\frac{2}{3}$

$\therefore\ y = -\frac{1}{3}x - \frac{2}{3}$

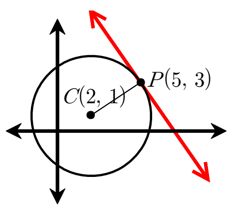
[EX13] Radius $\perp$ Tangent line.

Slope of CP $= \frac{3-1}{5-2} = \frac{2}{3}$

$\therefore$ Slope of the tangent line $= -\frac{3}{2}$

$Let\ \ y = -\frac{3}{2}x + b. \quad (5,\ 3)$

$3 = -\frac{15}{2} + b, \quad b = \frac{21}{2} \quad \Rightarrow \quad \therefore\ y = -\frac{3}{2}x + \frac{21}{2}$



[EX14] $x - 2y + 4 = 0 \qquad bx + y + c = 0$

$\Rightarrow y = \frac{1}{2}x + 2 \qquad \Rightarrow y = -bx - c$

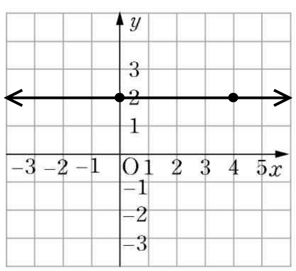
$\frac{1}{2} \times (-b) = -1$

$\therefore\ b = 2$

The two lines have the same y-intercept.

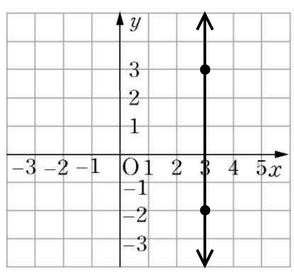
$2 = -c \qquad \therefore\ c = -2$

[EX15] ①



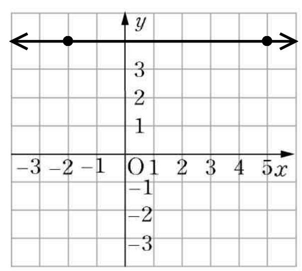
$y = 2$

②



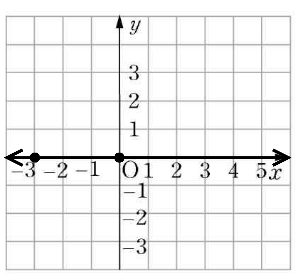
$x = 3$

③



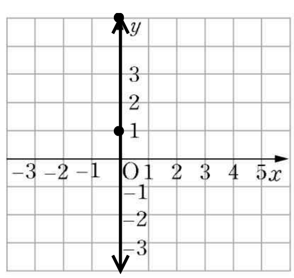
$y = 4$

④



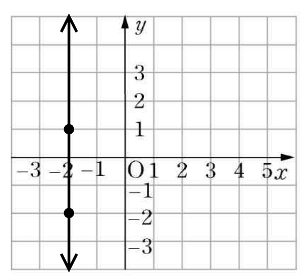
$y = 0$

⑤



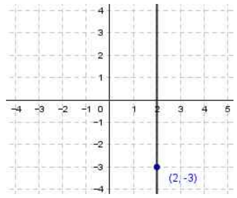
$x = 0$

⑥



$x = -2$

10. ①
$$\begin{array}{rl} & 4x+2y-2 = 0 \\ + & 3x-2y-12 = 0 \\ \hline & 7x \quad -14 = 0 \\ & x = 2 \end{array} \rightarrow \begin{array}{l} 2(2)+y-1 = 0 \\ y = -3 \\ \Rightarrow P(2,\ -3) \end{array}$$



$\therefore\ x = 2$

② $m_1 = \dfrac{5}{4} \quad \Rightarrow \quad m_2 = -\dfrac{4}{5}$

$\therefore\ y+3 = -\dfrac{4}{5}(x-2)$

$\left(or \quad y = -\dfrac{4}{5}x - \dfrac{7}{5}\right)$

11. Step 1. $m_1 = \dfrac{4}{3} \quad \Rightarrow \quad m_2 = -\dfrac{3}{4}$

perpendicular line through $(5,\ -1)$: $y+1 = -\dfrac{3}{4}(x-5)$

$y = -\dfrac{3}{4}x + \dfrac{15}{4} - 1$

$y = -\dfrac{3}{4}x + \dfrac{11}{4}$

Step 2. Intersection point: $\dfrac{4}{3}x + \dfrac{2}{3} = -\dfrac{3}{4}x + \dfrac{11}{4}$

Multiply by 12. $16x + 8 = -9x + 33$

$25x = 25$

$x = 1 \quad \rightarrow \quad y = \dfrac{4}{3}(1) + \dfrac{2}{3} = \dfrac{6}{3} = 2$

$\therefore\ H(1,\ 2)$

Step 3. Distance PH $= \sqrt{(1-5)^2 + (2+1)^2} = \sqrt{16+9} = 5$

12. Let the two numbers be x and y. $(x > y)$

(1) $x = y + 8$

(2) $x + 2y = \dfrac{1}{3}x - 8$

$3x + 6y = x - 24$

$2x + 6y = -24$

$x + 3y = -12$

(1)→(2): $(y+8) + 3y = -12$

$4y = -20$

$y = -5, \quad x = -5 + 8 = 3$

$\therefore\ 3,\ -5$

13. Let x and y be the tens digit and the ones digit, respectively.

(1) $y = 2x - 1$

(2) $10y + x = 10x + y + 18$

$-9x + 9y = 18$

$-x + y = 2$

(1)→(2): $-x + (2x - 1) = 2$

$x - 1 = 2$

$x = 3$

$y = 2(3) - 1$

$= 5 \qquad \therefore\ 35$

14. Let x be the number of adult tickets and y be the number of student tickets.

(1) $x + y = 120$

(2) $30x + 18y = 3060$

$5x + 3y = 510$

$$\begin{array}{rl} & 5x+3y = 510 \\ - & 3x+3y = 360 \\ \hline & 2x \quad = 150 \end{array}$$

$x = 75,\ y = 45$ $\quad \therefore$ 75 adult tickets and 45 student tickets.

15. Let x be the speed of the motorboat and y be the speed of the current.

	D	S	T
Up	60	$x-y$	4
Down	60	$x+y$	3

(1) $4(x-y) = 60$

$x - y = 15$

(2) $3(x+y) = 60$

$x + y = 20$

$$\begin{array}{rl} & x-y = 15 \\ + & x+y = 20 \\ \hline & 2x \quad = 35 \end{array}$$

$x = 17.5, \quad y = 2.5$

$\therefore$ The speed of the motorboat: 17.5 *km/h*

The speed of the current: 2.5 *km/h*

16. Let x be the time spent travelling by car and y be the time spent travelling by train.

	D	S	T
Car	$100x$	100	x
Train	$80y$	80	y

(1) $x + y = 5.5$

(2) $100x + 80y = 520$

$5x + 4y = 26$

$$\begin{array}{rl} & 5x+4y = 26 \\ - & 4x+4y = 22 \\ \hline & x \quad = 4,\ y = 1.5 \end{array}$$

$\therefore$ 4 *h* by car and 1.5 *h* by train.

17. Let x be the number of *mL* of 50% solution used and y be the number of *mL* of 95% solution used.

(1) $x + y = 900$

(2) $0.5x + 0.95y = 0.7 \cdot 900$

$0.5x + 0.95y = 630$

$x + 1.9y = 1260$

$$\begin{array}{rl} & x+1.9y = 1260 \\ - & x+\ \ y = 900 \\ \hline & 0.9y = 360 \end{array}$$

$y = 400, \quad x = 500$

$\therefore$ 500 *mL* of the 50% solution and 400 *mL* of the 95% solution.

18. ①+②: $2x + y = 3$ ④

②×2+③: $5x - y = 4$ ⑤

④+⑤: $7x = 7$

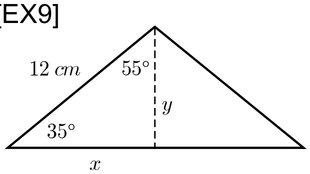
$\therefore\ x = 1$ → ④: $2(1) + y = 3$

$\therefore\ y = 1$ → ①: $(1) + 2(1) + z = 5$

$\therefore\ z = 2$

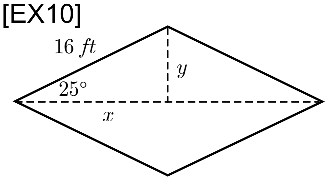
$\therefore\ (1,\ 1,\ 2)$

[EX9]



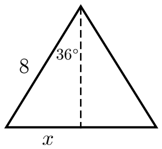
$$x = 12\cos 35° = 9.829\ldots$$
$$y = 12\sin 35° = 6.882\ldots$$
$$A = \frac{1}{2}(9.829\ldots)(6.882\ldots)\times 2$$
$$= 67.65\ldots = 67.7\ cm^2$$

[EX10]



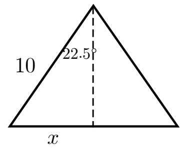
$$x = 16\cos 25° = 14.50\ldots$$
$$y = 16\sin 25° = 6.76\ldots$$
$$A = \frac{1}{2}(14.50\ldots)(6.76\ldots)\times 4$$
$$= 196.10\ldots = 196.1\ ft^2$$

[EX11] ①



$$\sin 36° = \frac{x}{8}$$
$$x = 8\sin 36° = 4.70\ldots$$
$$perimeter = 10\cdot x = 47.02\ldots$$
$$= 47.0\ cm$$

②

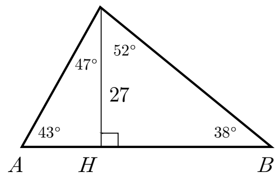


$$\sin 22.5° = \frac{x}{10}$$
$$x = 10\sin 22.5° = 3.82\ldots$$
$$perimeter = 16\cdot x = 61.22\ldots$$
$$= 61.2\ in$$

[EX12]

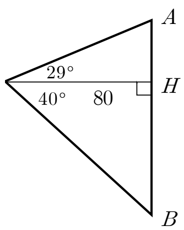
$$\tan 22° = \frac{AH}{x} \qquad \tan 42° = \frac{HB}{x}$$
$$AH = x\tan 22° \qquad HB = x\tan 42°$$
$$AH + HB = 200$$
$$x\tan 22° + x\tan 42° = 200$$
$$x(\tan 22° + \tan 42°) = 200$$
$$x = \frac{200}{\tan 22° + \tan 42°} = 153.32\ldots = 153.3\ m$$

[EX13]



$$\tan 47° = \frac{AH}{27} \qquad \tan 52° = \frac{HB}{27}$$
$$AH = 27\tan 47° \qquad HB = 27\tan 52°$$
$$AB = AH + HB$$
$$AB = 27\tan 47° + 27\tan 52°$$
$$= 63.51\ldots = 63.5\ m$$

[EX14]

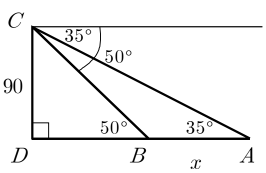


$$\tan 29° = \frac{AH}{80} \qquad \tan 40° = \frac{HB}{80}$$
$$AH = 80\tan 29° \qquad HB = 80\tan 40°$$
$$AB = AH + HB$$
$$= 80\tan 29° + 80\tan 40°$$
$$= 111.47\ldots = 111.5\ m$$

[EX15]

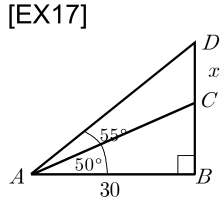
$$\angle ACD = 41° \qquad \tan 41° = \frac{AD}{x},\quad AD = x\tan 41°$$
$$\angle BCD = 35° \qquad \tan 35° = \frac{BD}{x},\quad BD = x\tan 35°$$
$$AD - BD = 30$$
$$x\tan 41° - x\tan 35° = 30$$
$$x(\tan 41° - \tan 35°) = 30$$
$$x = \frac{30}{\tan 41° - \tan 35°}$$
$$= 177.43\ldots$$
$$= 177.4\ m$$

[EX16]



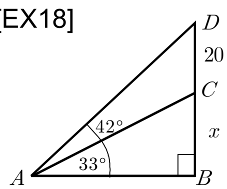
$$\angle ACD = 55° \qquad \tan 55° = \frac{AD}{90},\quad AD = 90\tan 55°$$
$$\angle BCD = 40° \qquad \tan 40° = \frac{BD}{90},\quad BD = 90\tan 40°$$
$$AB = AD - BD$$
$$= 90\tan 55° - 90\tan 40°$$
$$= 53.01\ldots = 53.0\ ft$$

[EX17]



$$\tan 55° = \frac{BD}{30} \qquad \tan 50° = \frac{BC}{30}$$
$$BD = 30\tan 55° \qquad BC = 30\tan 50°$$
$$x = BD - BC$$
$$= 30\tan 55° - 30\tan 50°$$
$$= 7.09\ldots = 7.1\ m$$

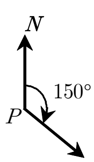
[EX18]



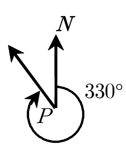
$$\angle ADB = 48° \qquad \angle ACB = 57°$$
$$\tan 48° = \frac{AB}{20+x} \qquad \tan 57° = \frac{AB}{x}$$
$$AB = (20+x)\tan 48° \ (1) \qquad AB = x\tan 57° \ (2)$$

$$(20+x)\tan 48° = x\tan 57°$$
$$20\tan 48° + x\tan 48° = x\tan 57°$$
$$20\tan 48° = x\tan 57° - x\tan 48°$$
$$20\tan 48° = x(\tan 57° - \tan 48°) \rightarrow x = \frac{20\tan 48°}{\tan 57° - \tan 48°}$$
$$= 51.74\ldots = 51.7\ ft$$

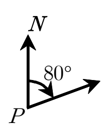
[EX19] ①



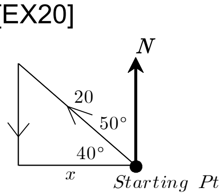
②



③

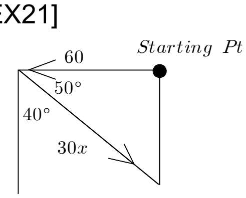


[EX20]



$$10\ mph\cdot 2h = 20\ miles$$
$$\cos 40° = \frac{x}{20}$$
$$x = 20\cos 40°$$
$$= 15.32\ldots = 15.3\ miles$$

[EX21]



$$20\ km/h\cdot 3h = 60\ km$$
$$30\ km/h\cdot x\,h = 30x\ km$$
$$\cos 50° = \frac{60}{30x} = \frac{2}{x}$$
$$x = \frac{2}{\cos 50°} = 3.11\ldots = 3.1\,hr$$

At all levels of mathematics, the best way to learn new concepts is by solving numerous questions. Through this Pre-calculus 10 workbook, students master the mathematical foundations required to succeed in Calculus.
Following a short review of the fundamental concepts and guided examples, students are challenged with exercise questions ranging in difficulty. There are over 400 guided examples and 3300 questions, with full solutions, because practice makes perfect.

www.infinitechallenge.ca

ISBN 978-1-0690212-0-5

9 781069 021205 >